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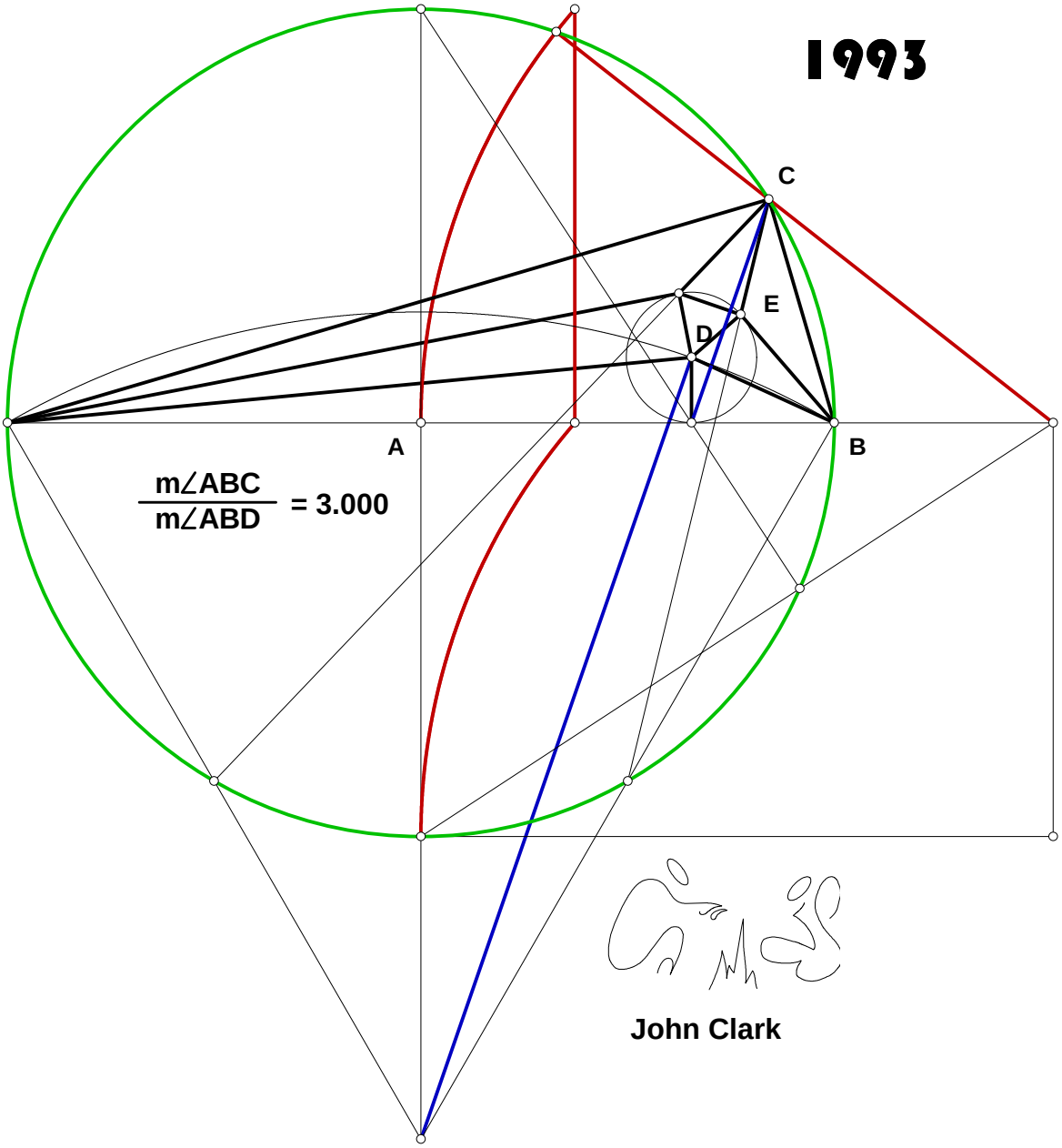
1993



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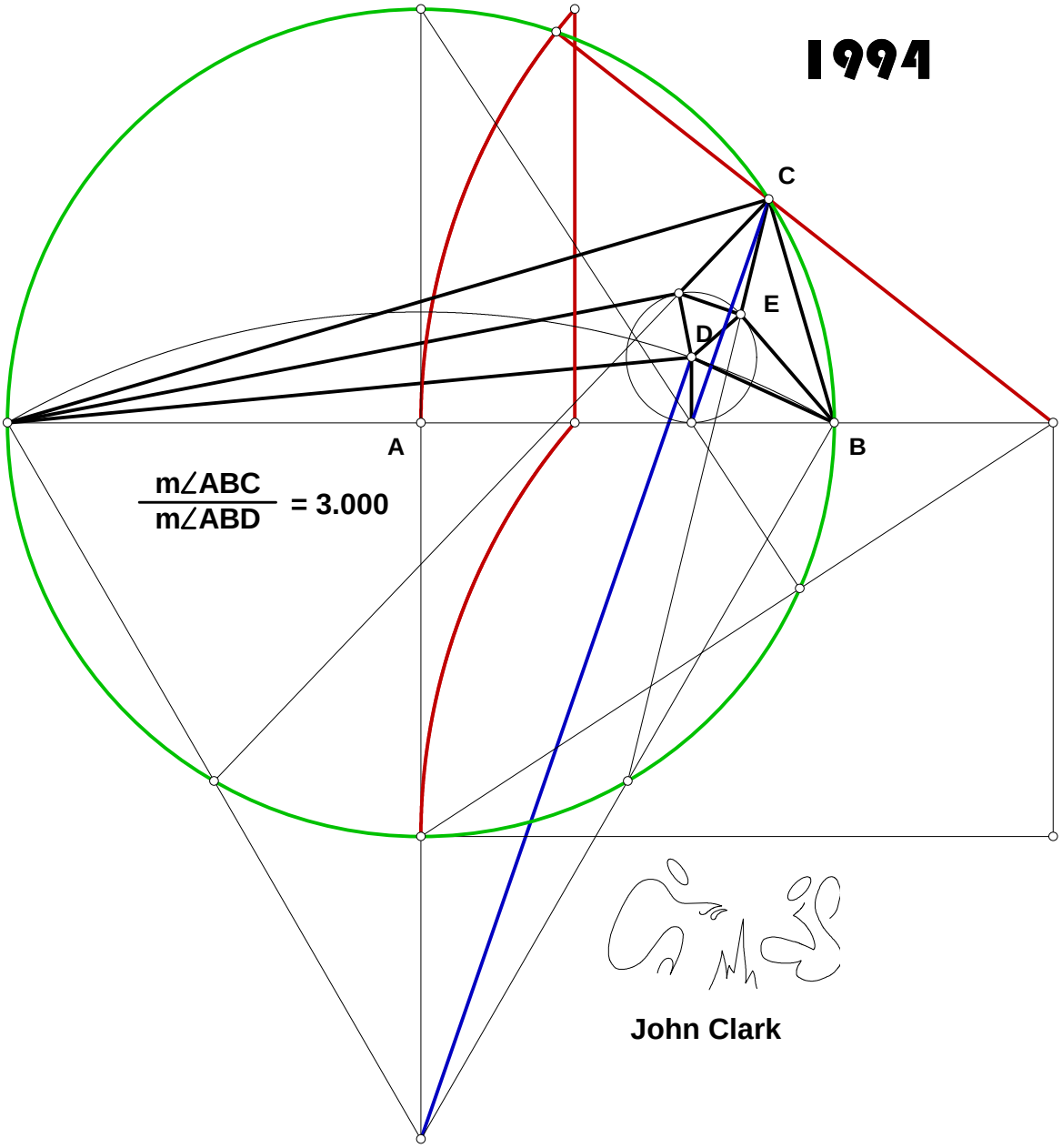
1994



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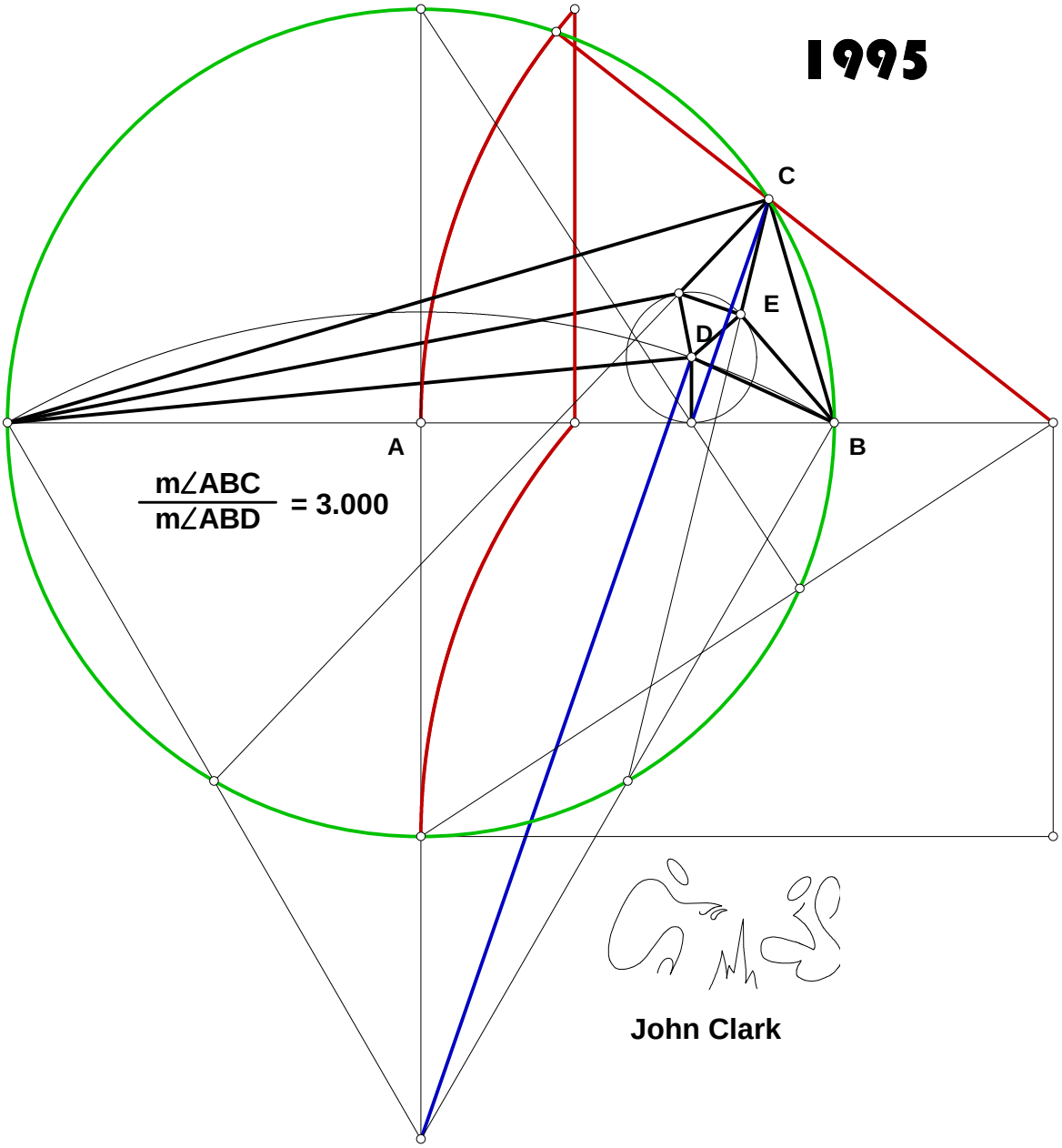
1995



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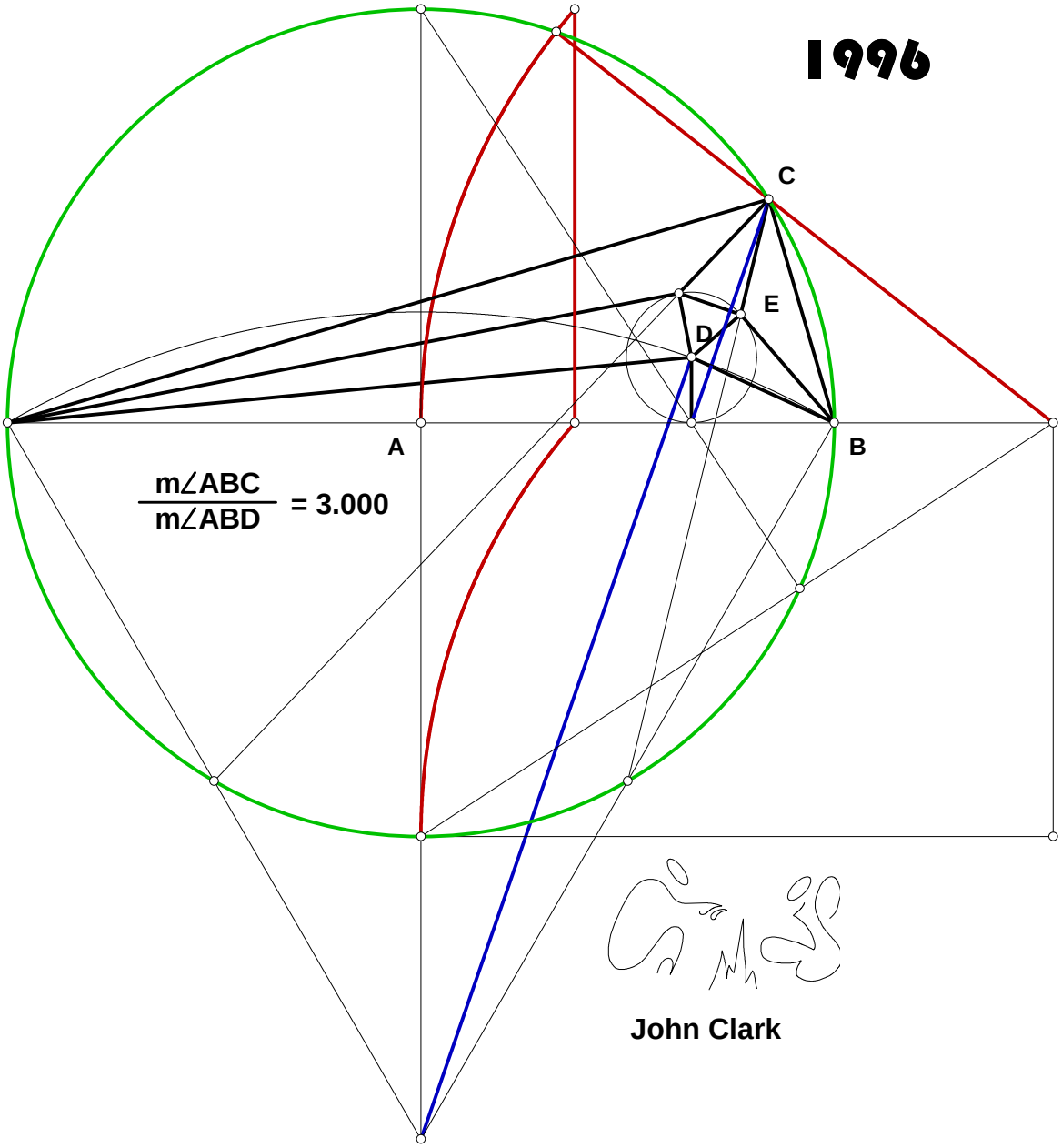
1996



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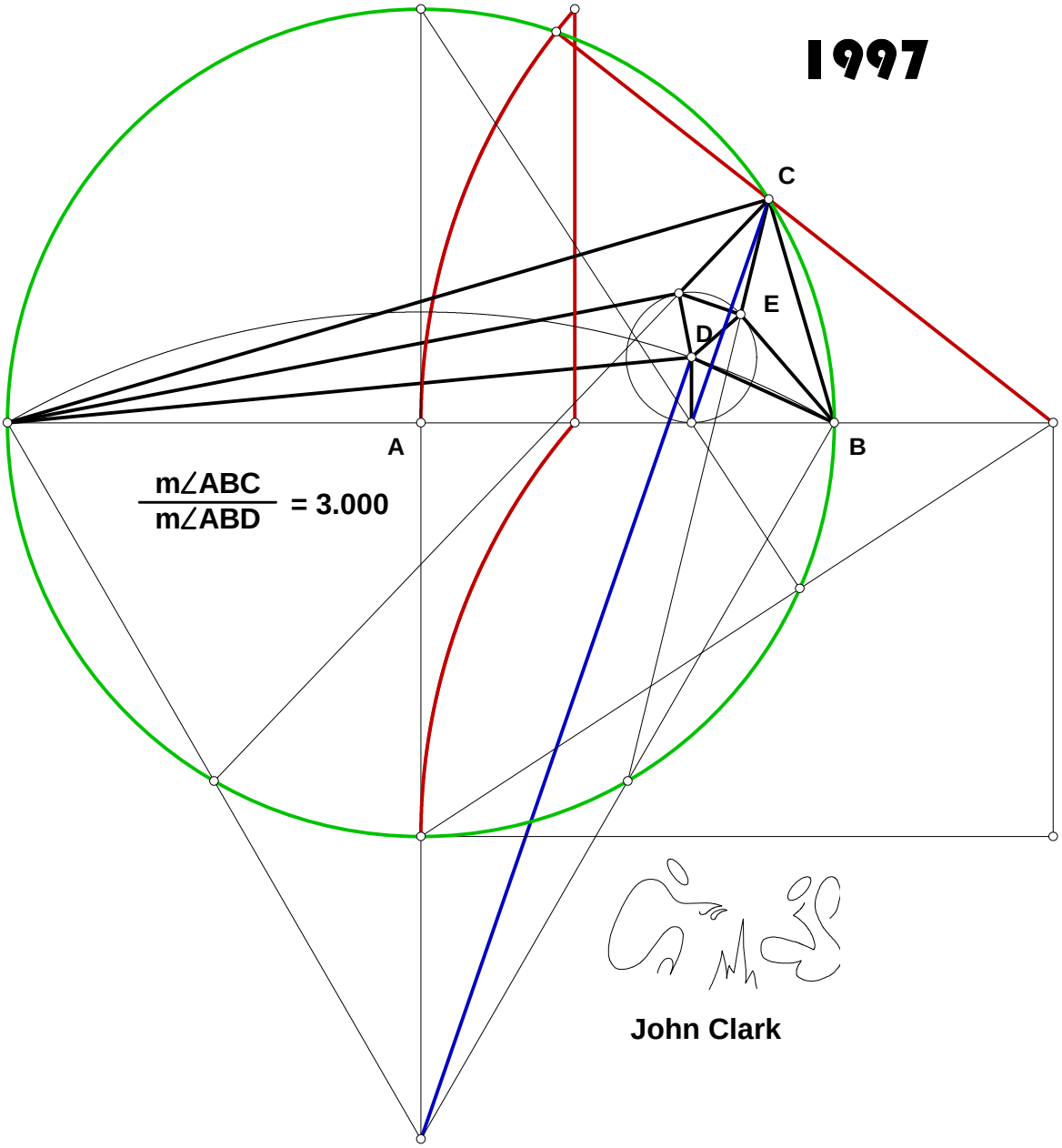
1997



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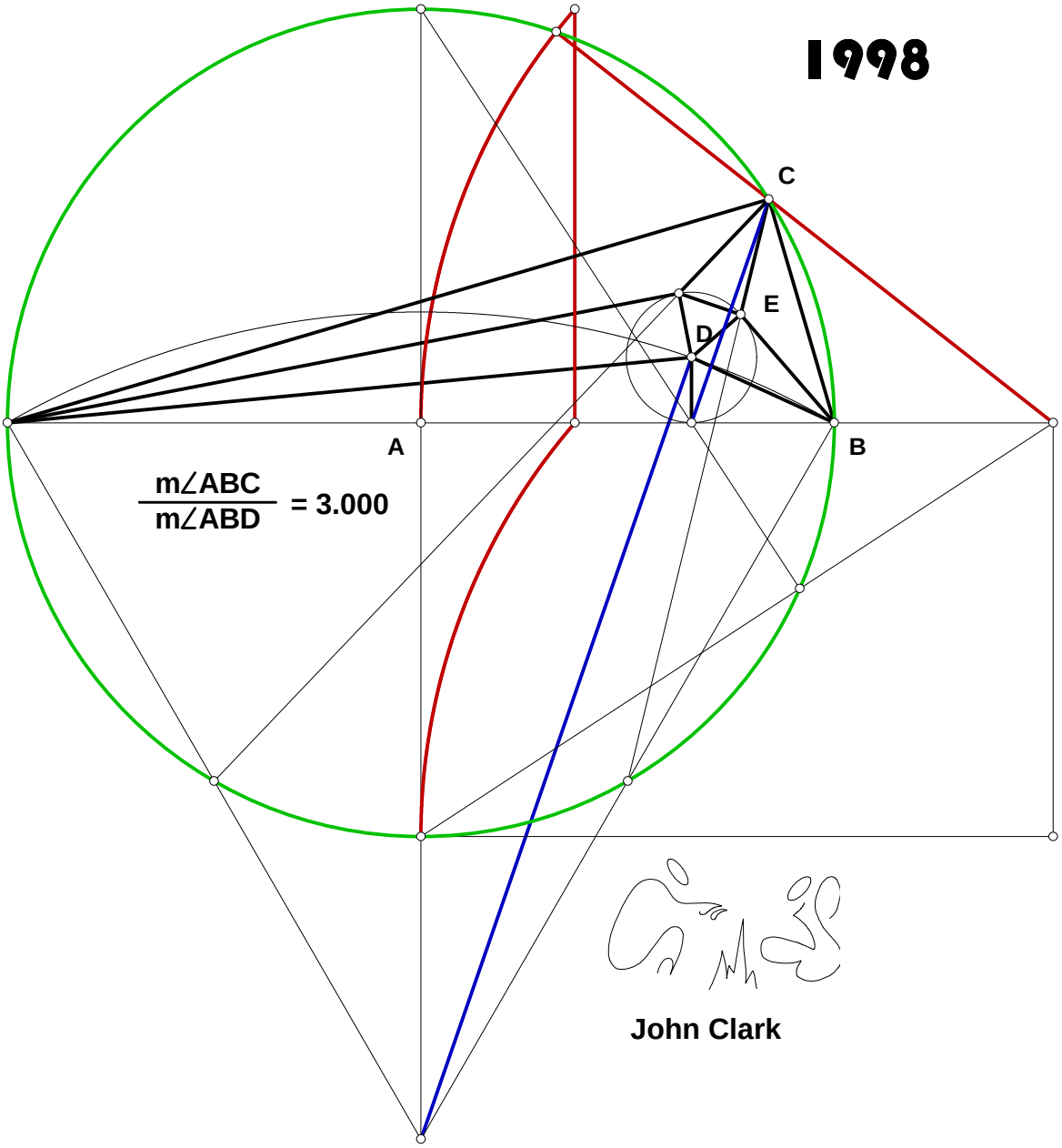

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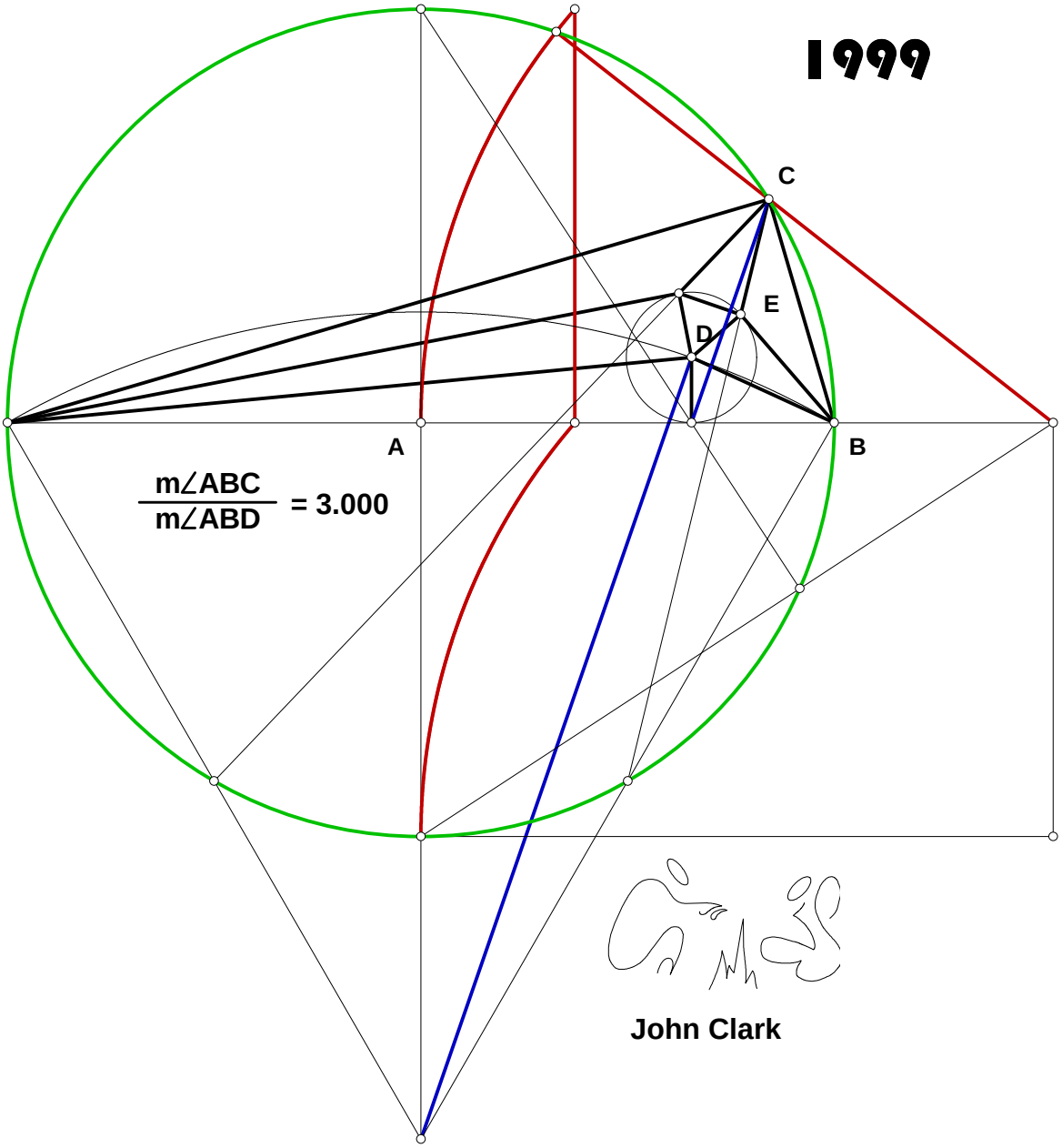

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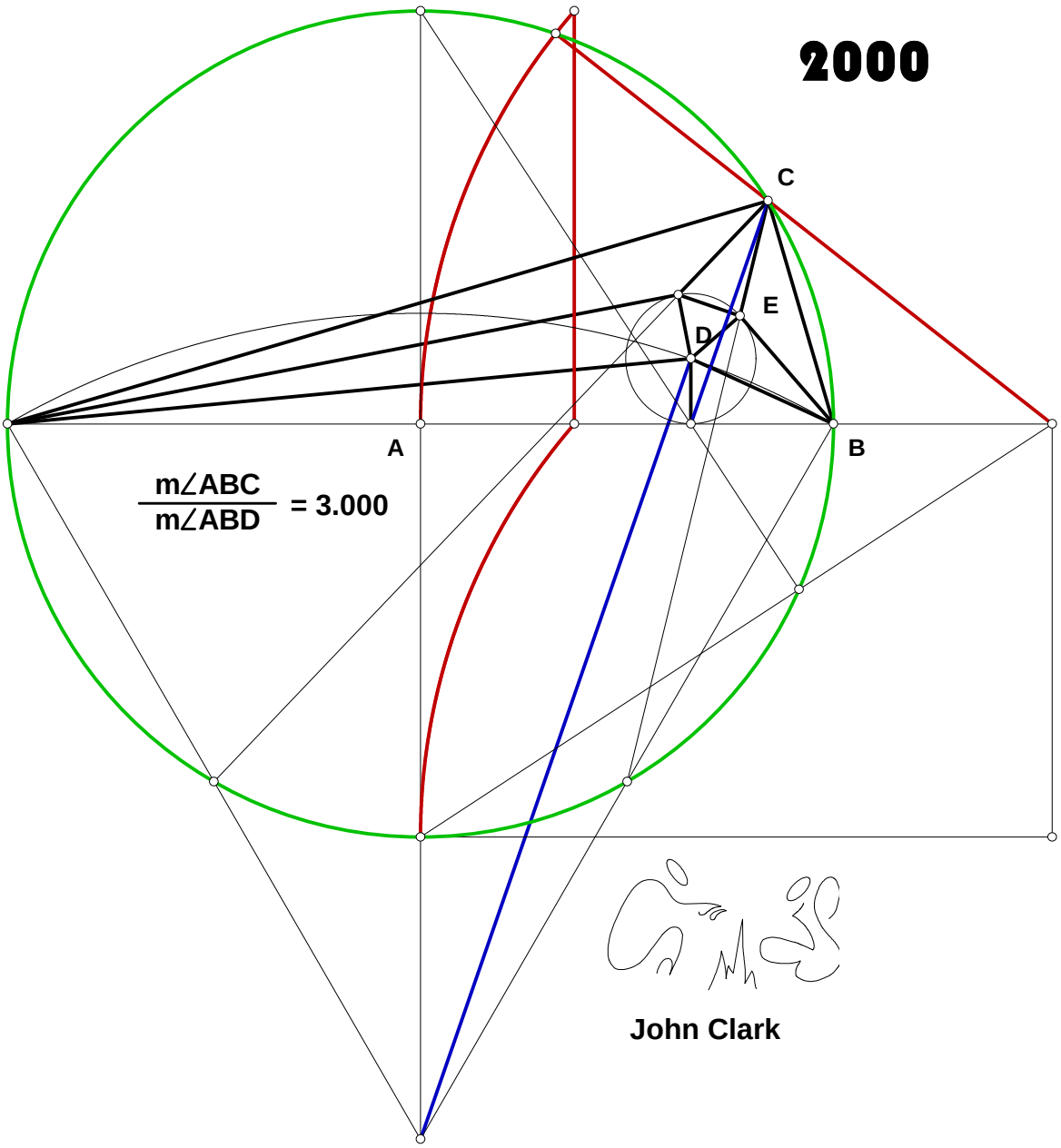

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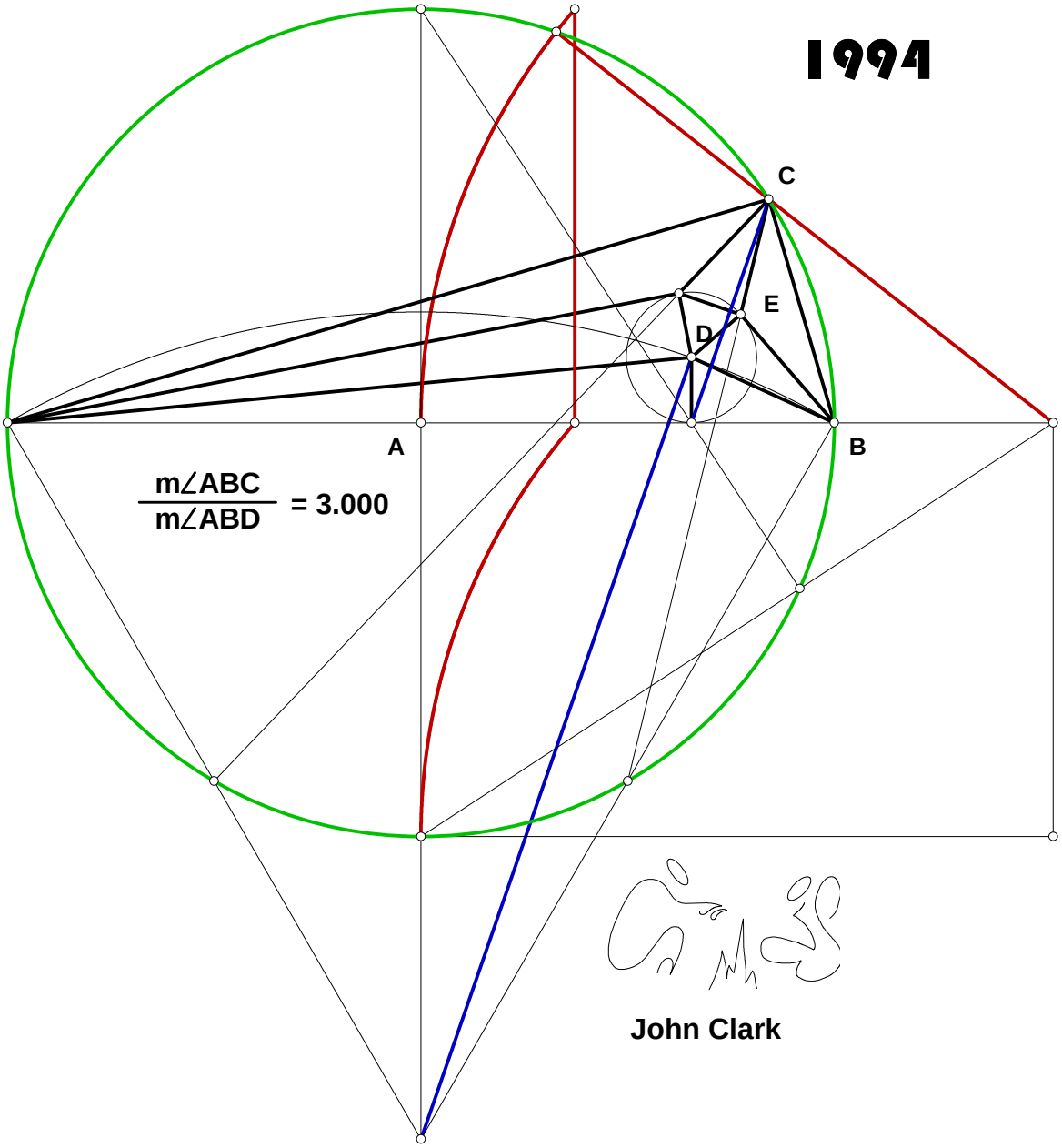

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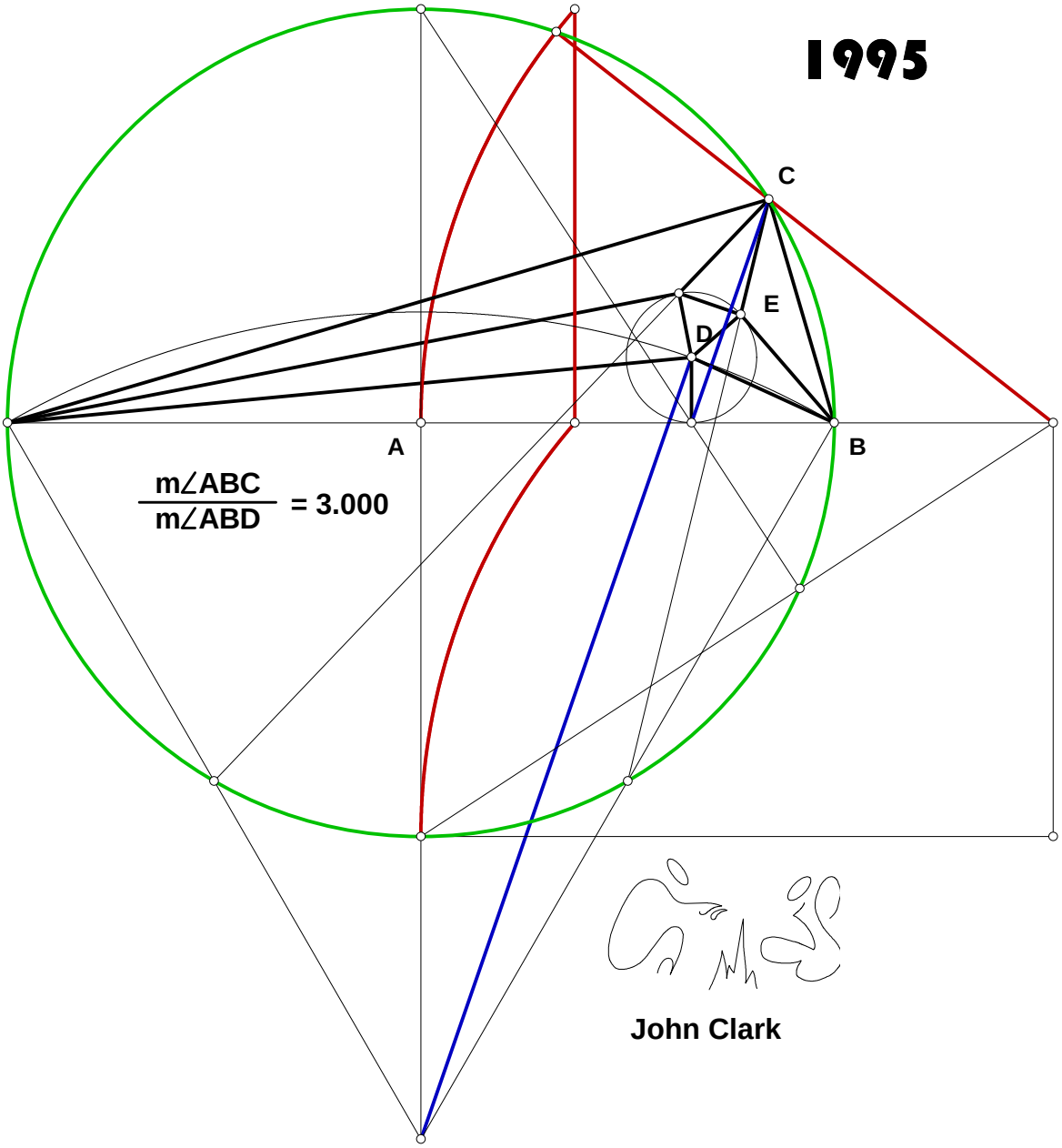

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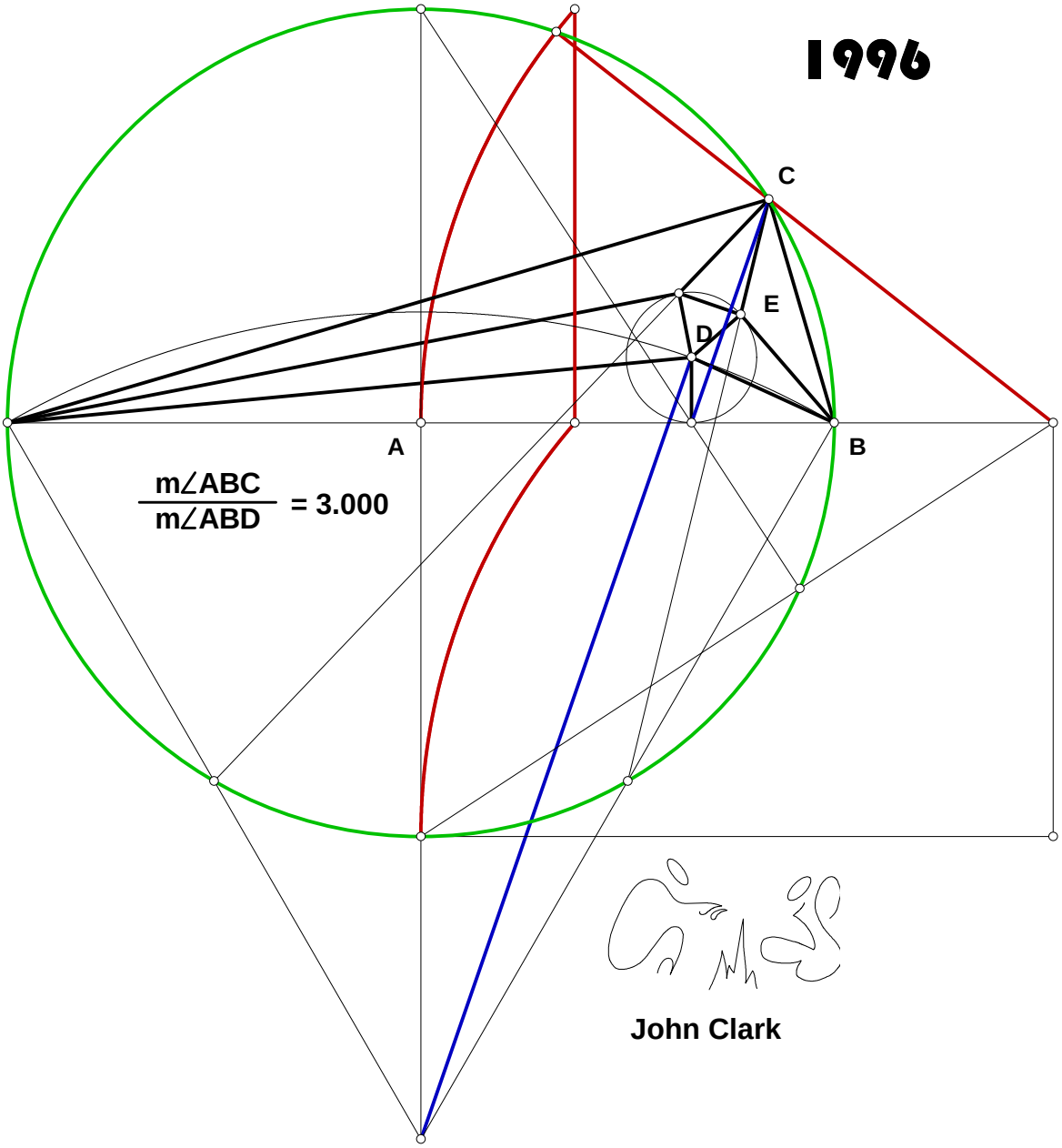

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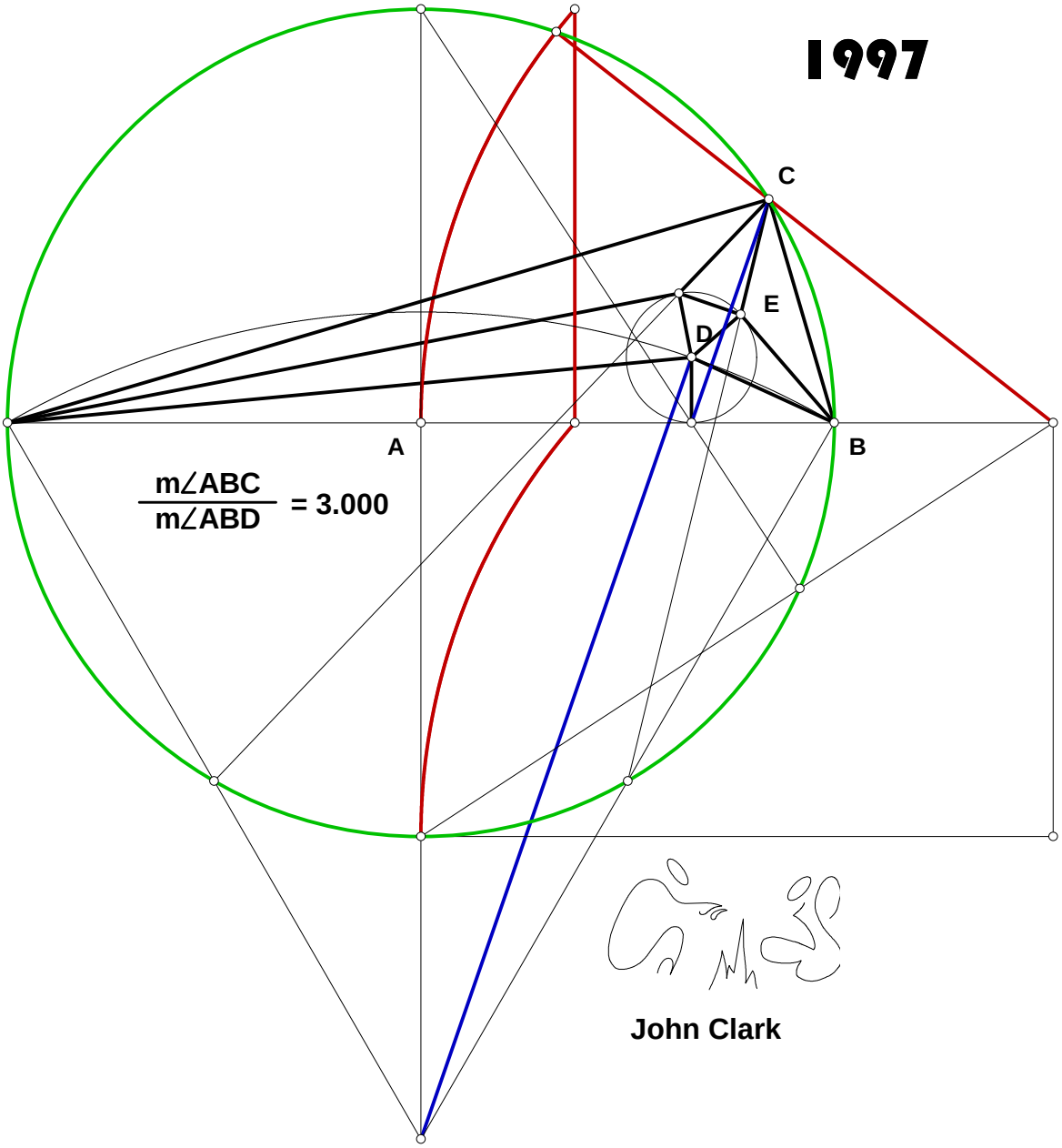

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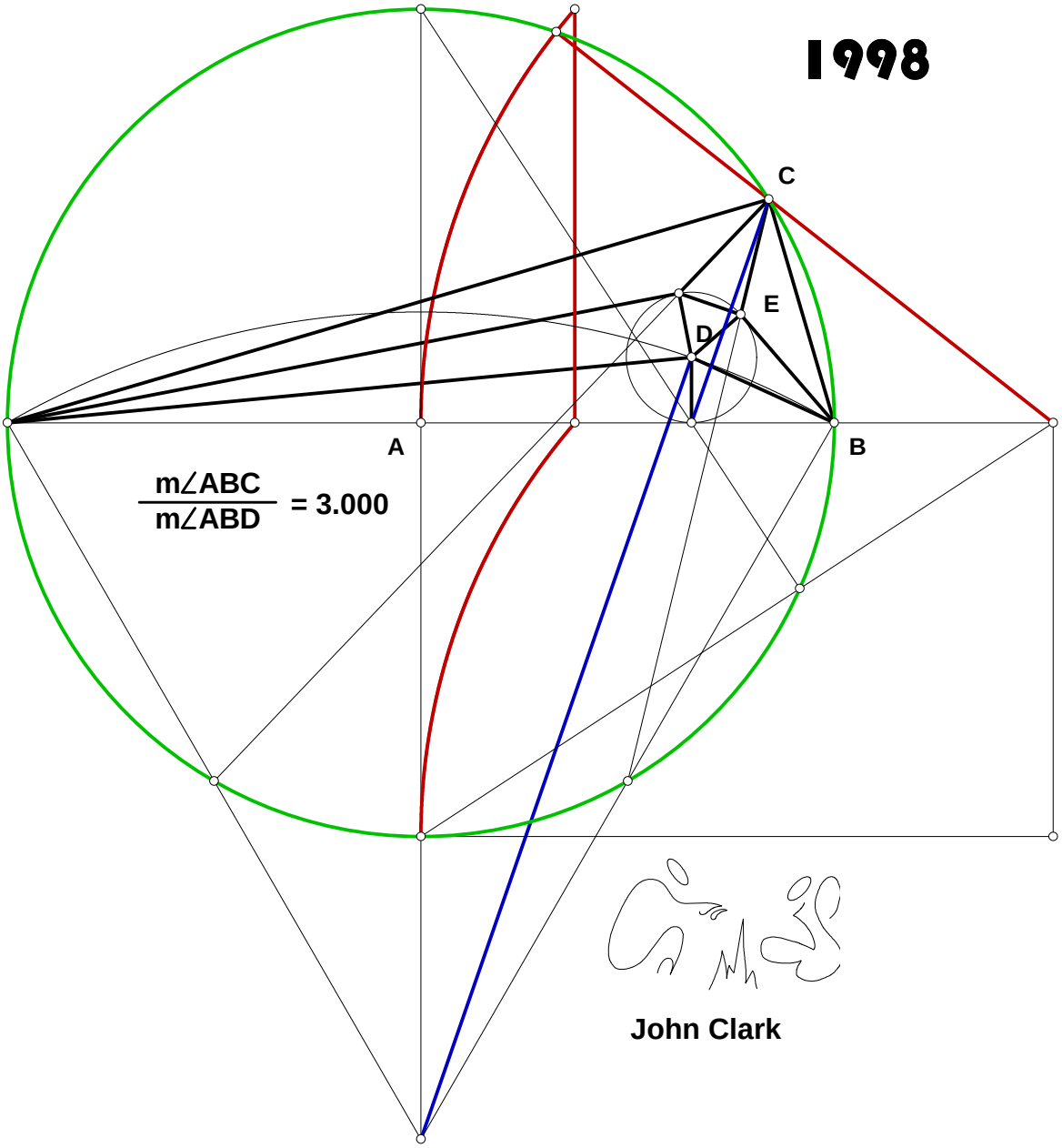

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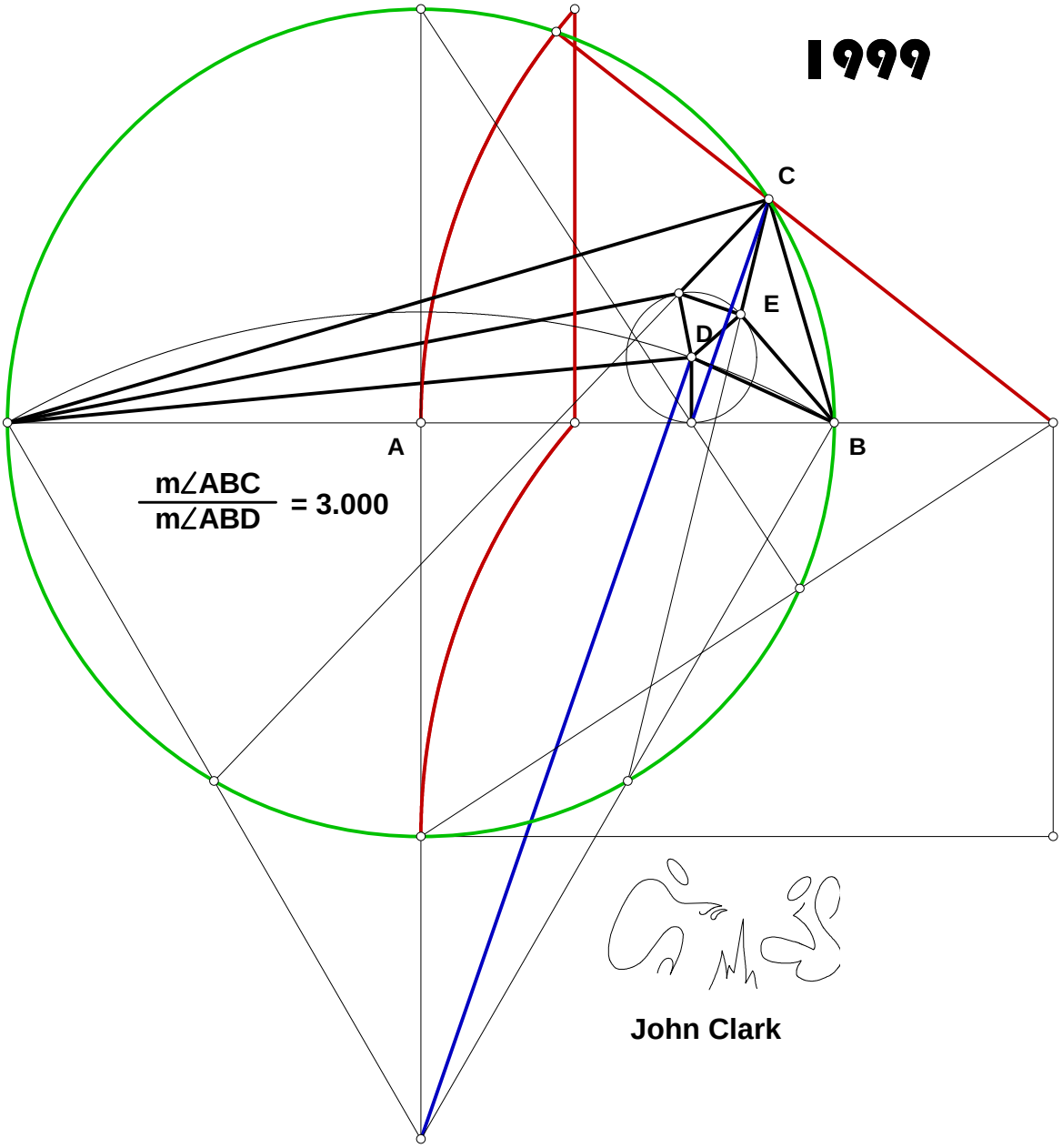

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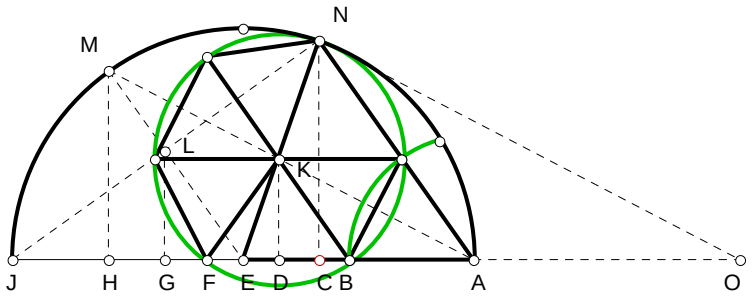
11/11/93 The Archamedian Paper Trisector



$N := 4$ $AJ := 1$ $AE := \frac{AJ}{2}$ $EJ := AE$ $EN := AE$ $EM := AE$ $AC := \frac{AJ}{N}$

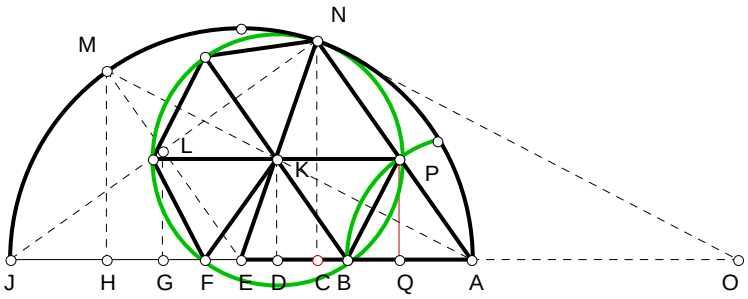
$CJ := AJ - AC$ $CN := \sqrt{AC \cdot CJ}$ $JN := \sqrt{CN^2 + CJ^2}$ $JL := \frac{JN}{2}$ $GL := \frac{CN}{2}$ $GJ := \frac{CJ}{2}$

$EG := EJ - GJ$ $EL := \sqrt{EG^2 + GL^2}$ $EH := \frac{EG \cdot EM}{EL}$ $HM := \frac{GL \cdot EM}{EL}$ $AH := AE + EH$

$CO := \frac{AH \cdot CN}{HM}$ $CE := AE - AC$ $EO := CO + CE$ $EK := \frac{EN \cdot AE}{EO}$ $DE := \frac{CE \cdot AE}{EO}$ $DK := \frac{CN \cdot EK}{EN}$

$AD := AE - DE$ $KN := EN - EK$ $BK := KN$ $BD := \sqrt{BK^2 - DK^2}$ $AB := AD - BD$

$AB - BK = 0$ $AB = 0.25$ **If PK is parallel to AJ, then . . . .**



$AN := 2 \cdot EL$ $AP := AB$ $PQ := \frac{CN \cdot AP}{AN}$ $PQ - DK = 0$

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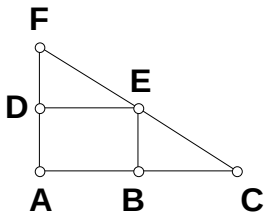
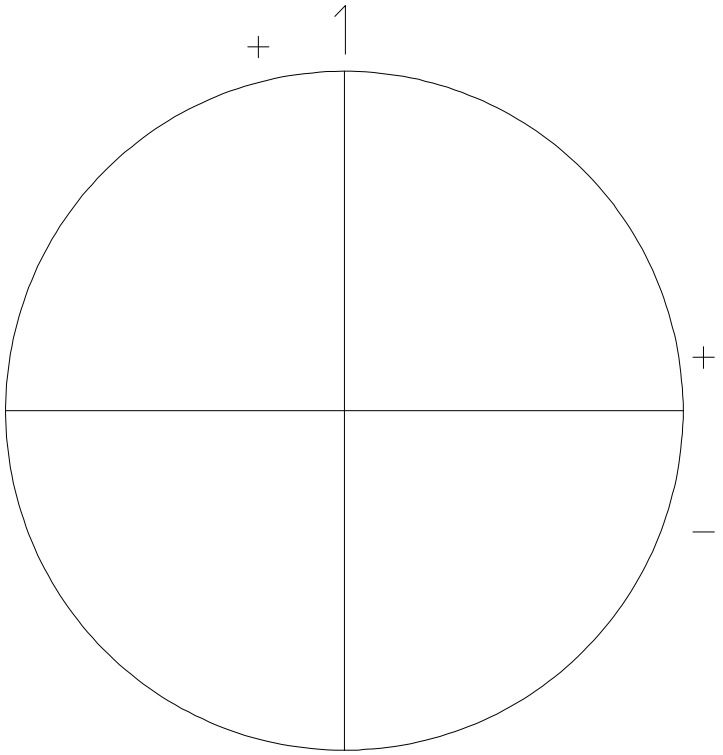
Given AB, AF, BE, what is EF?



$N_1 := 2 \quad N_2 := 3 \quad N_3 := 5 \quad AF := N_1 \quad BE := N_2 \quad AB := N_3$

$AD := BE \quad DE := AB \quad DF := AF - AD \quad EF := \left(DF^2 + DE^2\right)^{\frac{1}{2}}$

$EF - \sqrt{N_1^2 - 2 \cdot N_1 \cdot N_2 + N_2^2 + N_3^2} = 0$



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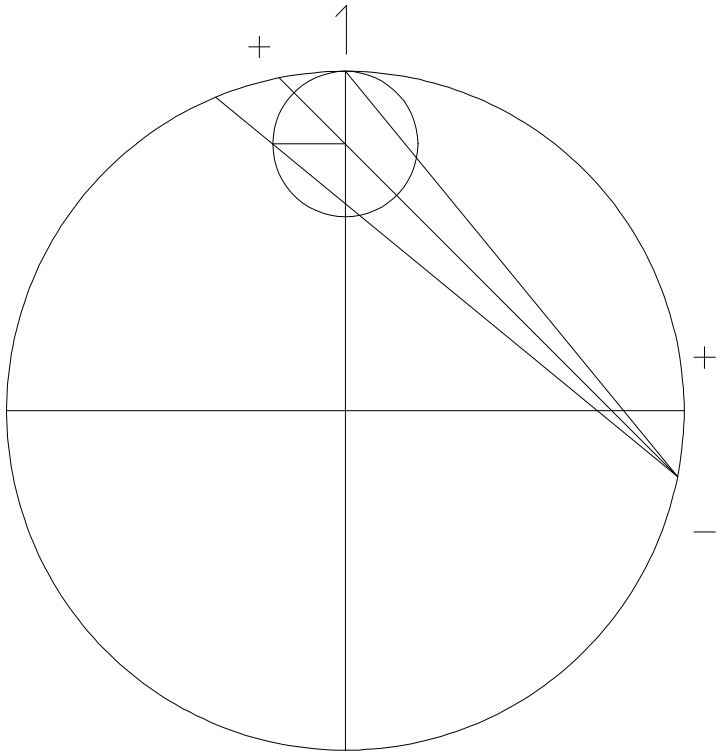
Archamedian Trisection Revisited.

I am curious as to why the Archamedian trisection is still taught the way it was given so long ago and the idea from which it derives is not in any of my books.

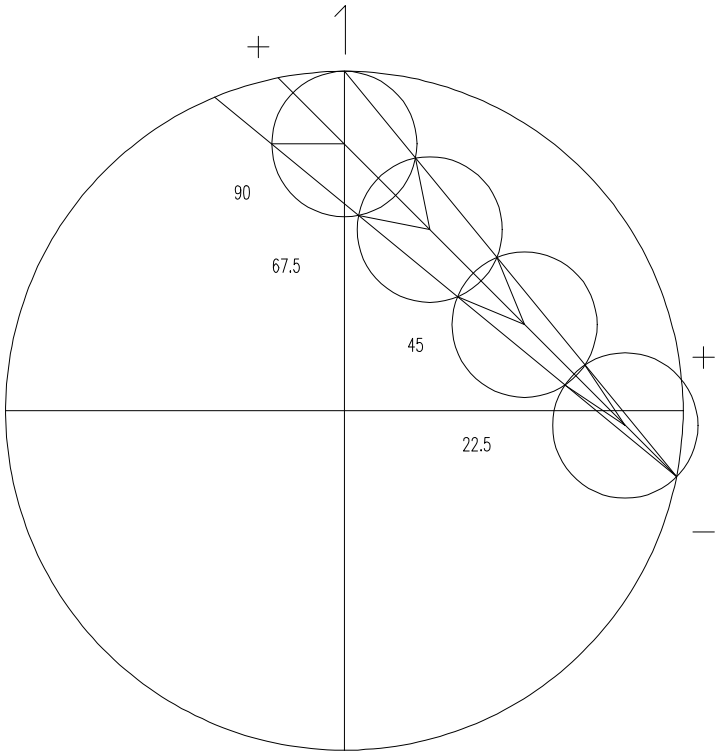
I am going to use a circle to demonstrate some principles concerning angles. To help log the results in mathematical terms, I will divide the circle into quadrants and label them either plus or minus. The top is labeled one.



The circle is divided into 1/8 segments of 90 degrees for each quadrant. I start at the top. Since the angle at the perimeter is 1/2 of the angle at the center, when I start the figure I use twice the arc segment for the angle I am working with. I have marked some quadrants with plus and minus and have found that for the figure, I would say that I have 1 + 1/8 - 1/8. From this I have learned that my starting angle of the smaller circle will be 90 degrees.

$$\frac{90}{8} = 11.25$$

$$90 + \frac{90}{8} - \frac{90}{8} = 90 \qquad \left(1 + \frac{1}{8} - \frac{1}{8}\right) \cdot 90 = 90$$



$$B := 1 + \frac{1}{8} - \frac{1}{8} \qquad B = 1$$

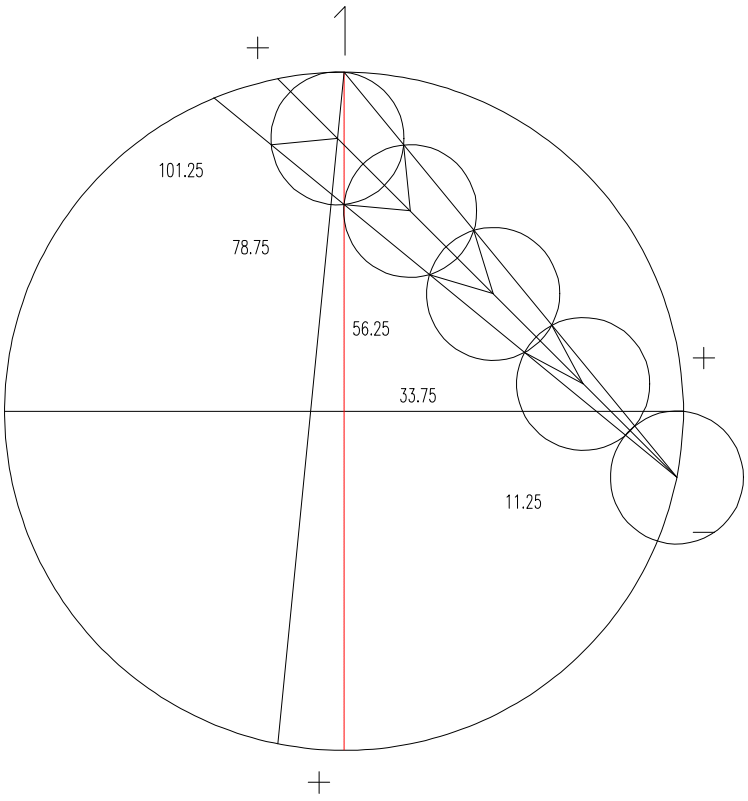
$$\frac{B \cdot 4}{4} \cdot 90 = 90 \qquad \frac{B \cdot 3}{4} \cdot 90 = 67.5$$

$$\frac{B \cdot 2}{4} \cdot 90 = 45 \qquad \frac{B}{4} \cdot 90 = 22.5$$

$8 + 1 - 1 = 8$	$8 \cdot 11.25 = 90$
$8 + 1 - 1 - 2 = 6$	$6 \cdot 11.25 = 67.5$
$8 + 1 - 1 - 2 - 2 = 4$	$4 \cdot 11.25 = 45$
$8 + 1 - 1 - 2 - 2 - 2 = 2$	$2 \cdot 11.25 = 22.5$
$8 + 1 - 1 - 2 - 2 - 2 - 2 = 0$	

$$\mathrm{mod}(8 + 1 - 1, 2) = 0$$

I have added another plus to a quadrant at the bottom of the figure.



$$B := 1 + \frac{1}{8} + \frac{1}{8} - \frac{1}{8} \qquad B = 1.125 \qquad \frac{9}{8} = 1.125$$

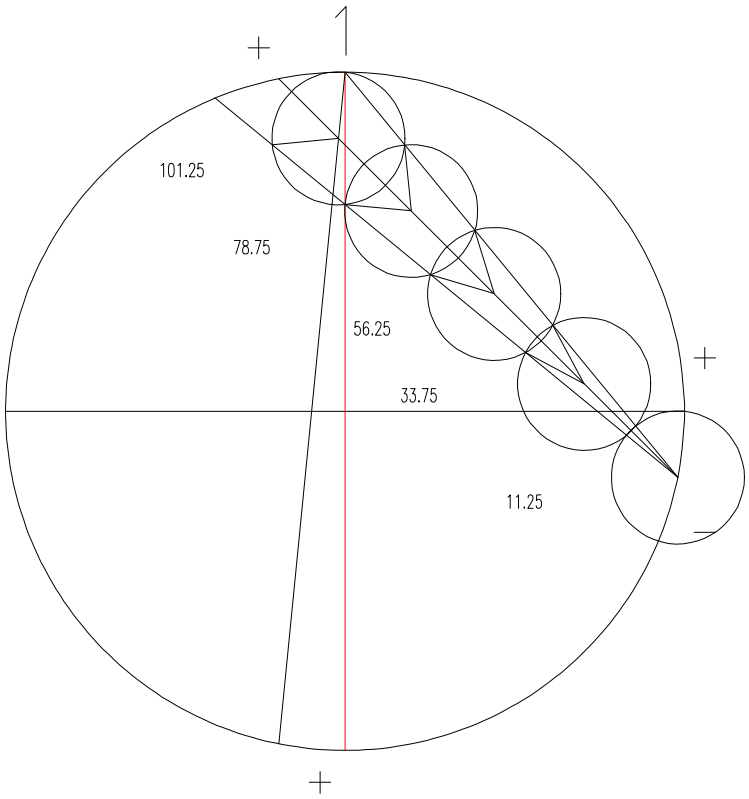
$$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25 \qquad \frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$$

$$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25 \qquad \frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$$

$$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$$

$8 + 1 + 1 - 1 = 9$	$9 \cdot 11.25 = 101.25$
$8 + 1 + 1 - 1 - 2 = 7$	$7 \cdot 11.25 = 78.75$
$8 + 1 + 1 - 1 - 2 - 2 = 5$	$5 \cdot 11.25 = 56.25$
$8 + 1 + 1 - 1 - 2 - 2 - 2 = 3$	$3 \cdot 11.25 = 33.75$
$8 + 1 + 1 - 1 - 2 - 2 - 2 - 2 = 1$	$1 \cdot 11.25 = 11.25$

$$\mathrm{mod}(8 + 1 + 1 - 1, 2) = 1$$



$$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{1}{8} \qquad B = 1.125 \qquad \frac{9}{8} = 1.125$$

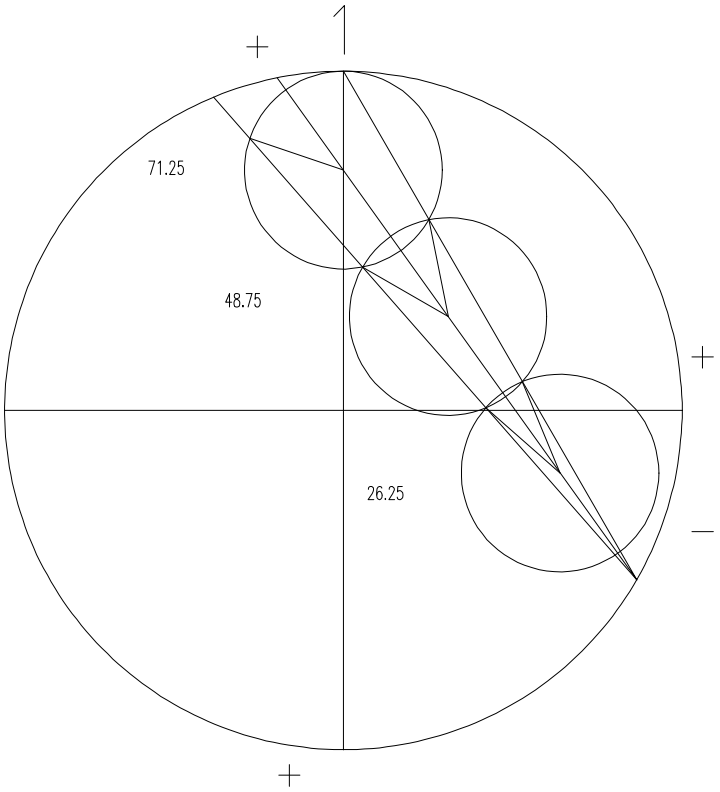
$$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25 \qquad \frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$$

$$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25 \qquad \frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$$

$$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$$

$8 + 1 = 9$	$9 \cdot 11.25 = 101.25$
$8 + 1 - (1 \cdot 2) = 7$	$7 \cdot 11.25 = 78.75$
$8 + 1 - (2 \cdot 2) = 5$	$5 \cdot 11.25 = 56.25$
$8 + 1 - (3 \cdot 2) = 3$	$3 \cdot 11.25 = 33.75$
$8 + 1 - (4 \cdot 2) = 1$	$1 \cdot 11.25 = 11.25$

$$\mathbf{mod}(8 + 1, 2) = 1$$



$$B := 1 + \frac{3}{24} - \frac{8}{24} \qquad B = 0.7917 \qquad \frac{19}{24} = 0.7917$$

$$\frac{B \cdot 3.1666}{3.1666} \cdot 90 = 71.25 \qquad \frac{B \cdot 2.1666}{3.1666} \cdot 90 = 48.7495$$

$$\frac{B \cdot 1.16666}{3.16666} \cdot 90 = 26.2499 \qquad \frac{B \cdot .166666}{3.166666} \cdot 90 = 3.75$$

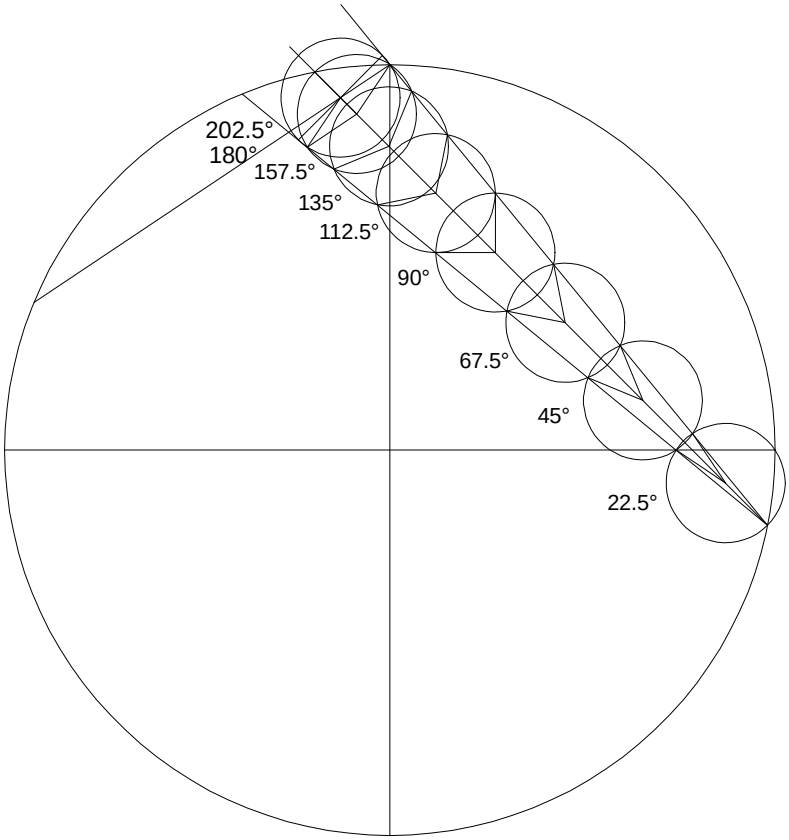
$(24 + 3) - 8 = 19$	$19 \cdot 3.75 = 71.25$
$(24 + 3) - 8 - (1 \cdot 6) = 13$	$13 \cdot 3.75 = 48.75$
$(24 + 3) - 8 - (2 \cdot 6) = 7$	$7 \cdot 3.75 = 26.25$
$(24 + 3) - 8 - (3 \cdot 6) = 1$	$1 \cdot 3.75 = 3.75$

$$\mathbf{mod}(24 + 3 - 8, 2) = 1$$

$$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{10}{8} \qquad B = 2.25 \qquad \frac{18}{8} = 2.25$$

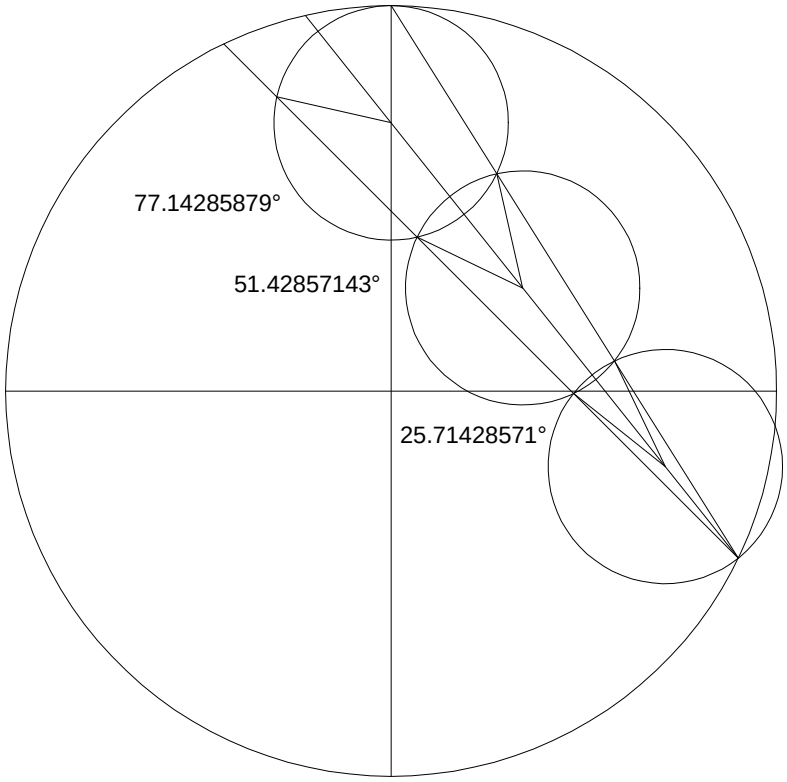
$$\frac{B \cdot 9}{9} \cdot 90 = 202.5 \qquad \frac{B \cdot 8}{9} \cdot 90 = 180$$

$$\frac{B \cdot 7}{9} \cdot 90 = 157.5 \qquad \frac{B \cdot .6}{9} \cdot 90 = 13.5$$



$8 + 1 - 1 + 10 = 18$	$18 \cdot 11.25 = 202.5$
$8 + 1 - 1 + 10 - (2 \cdot 1) = 16$	$16 \cdot 11.25 = 180$
$8 + 1 - 1 + 10 - (2 \cdot 2) = 14$	$14 \cdot 11.25 = 157.5$
$8 + 1 - 1 + 10 - (2 \cdot 3) = 12$	$12 \cdot 11.25 = 135$
$8 + 1 - 1 + 10 - (2 \cdot 4) = 10$	$10 \cdot 11.25 = 112.5$
$8 + 1 - 1 + 10 - (2 \cdot 5) = 8$	$8 \cdot 11.25 = 90$
$8 + 1 - 1 + 10 - (2 \cdot 6) = 6$	$6 \cdot 11.25 = 67.5$
$8 + 1 - 1 + 10 - (2 \cdot 7) = 4$	$4 \cdot 11.25 = 45$
$8 + 1 - 1 + 10 - (2 \cdot 8) = 2$	$2 \cdot 11.25 = 22.5$

$$\mathbf{mod}((8 + 1 - 1) + 10, 2) = 0$$



$$B := 1 + \frac{1}{7} - \frac{2}{7} \qquad B = 0.8571 \qquad \frac{6}{7} = 0.8571$$

$$\frac{B \cdot 6}{6} \cdot 90 = 77.1429 \qquad \frac{B \cdot 4}{6} \cdot 90 = 51.4286$$

$$\frac{B \cdot 2}{6} \cdot 90 = 25.7143 \qquad c := \frac{90}{7}$$

$7 + 1 - (1 \cdot 2) = 6$	$6 \cdot c = 77.1429$
$7 + 1 - (2 \cdot 2) = 4$	$4 \cdot c = 51.4286$
$7 + 1 - (3 \cdot 2) = 2$	$2 \cdot c = 25.7143$

$$\mathbf{mod}(7 + 1 - 2, 2) = 0$$

$$B := 1 + \frac{1}{7} - \frac{1}{7} \qquad B = 1 \qquad \frac{7}{7} = 1$$

$$\frac{B \cdot 7}{7} \cdot 90 = 90 \qquad \frac{B \cdot 5}{7} \cdot 90 = 64.2857$$

$$\frac{B \cdot 3}{7} \cdot 90 = 38.5714 \qquad \frac{B \cdot 1}{7} \cdot 90 = 12.8571$$

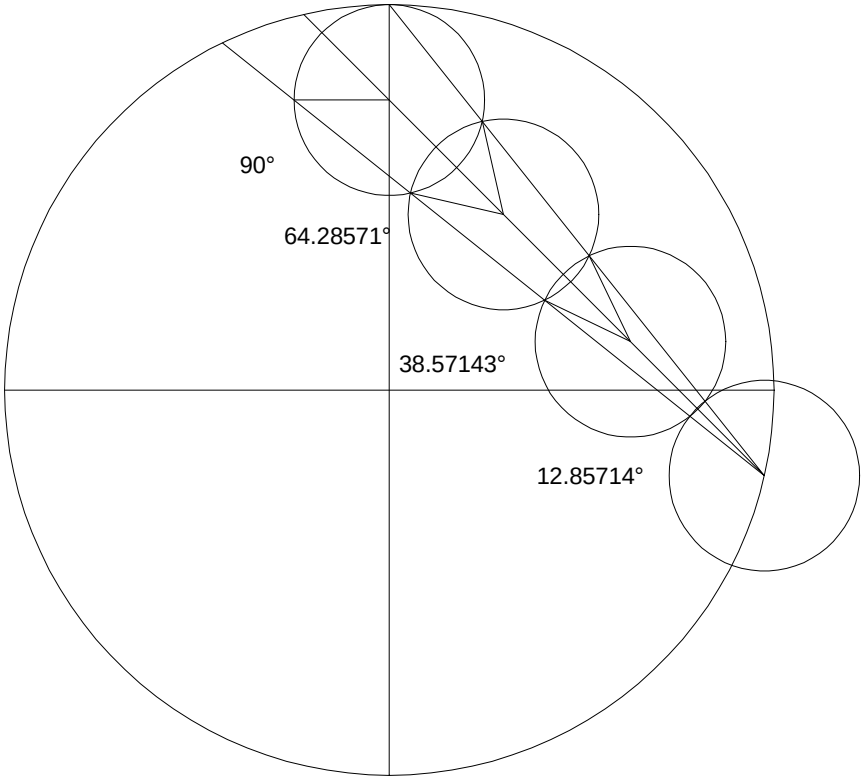
$$7 + 1 - 1 = 7 \qquad 7 \cdot c = 90$$

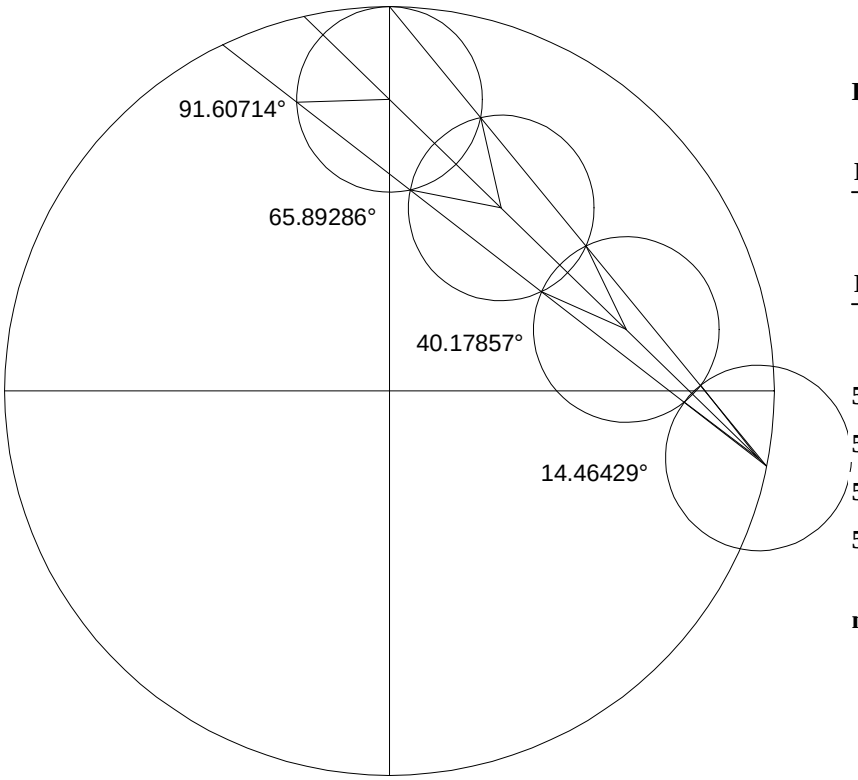
$$7 + 1 - 1 - (1 \cdot 2) = 5 \qquad 5 \cdot c = 64.2857$$

$$7 + 1 - 1 - (2 \cdot 2) = 3 \qquad 3 \cdot c = 38.5714$$

$$7 + 1 - 1 - (3 \cdot 2) = 1 \qquad 1 \cdot c = 12.8571$$

$$\mathbf{mod}(7 + 1 - 1, 2) = 1$$





$$B := 1 + \frac{8}{56} - \frac{7}{56} \qquad B = 1.0179$$

$$\frac{B \cdot 57}{57} \cdot 90 = 91.6071 \qquad \frac{B \cdot 41}{57} \cdot 90 = 65.8929$$

$$\frac{B \cdot 25}{57} \cdot 90 = 40.1786 \qquad c := \frac{90}{56}$$

$$56 + 8 - 7 = 57 \qquad 57 \cdot c = 91.6071$$

$$56 + 8 - 7 - (1 \cdot 16) = 41 \qquad 41 \cdot c = 65.8929$$

$$56 + 8 - 7 - (2 \cdot 16) = 25 \qquad 25 \cdot c = 40.1786$$

$$56 + 8 - 7 - (3 \cdot 16) = 9 \qquad 9 \cdot c = 14.4643$$

$$\mathbf{mod}(56 + 8 - 7, 16) = 9$$

$$B := 1 + \frac{1}{7} - \frac{1}{7} \qquad B = 1$$

$$\frac{B \cdot 7}{7} \cdot 90 = 90 \qquad \frac{B \cdot 5}{7} \cdot 90 = 64.2857$$

$$\frac{B \cdot 3}{7} \cdot 90 = 38.5714 \qquad \frac{B \cdot 1}{7} \cdot 90 = 12.8571$$

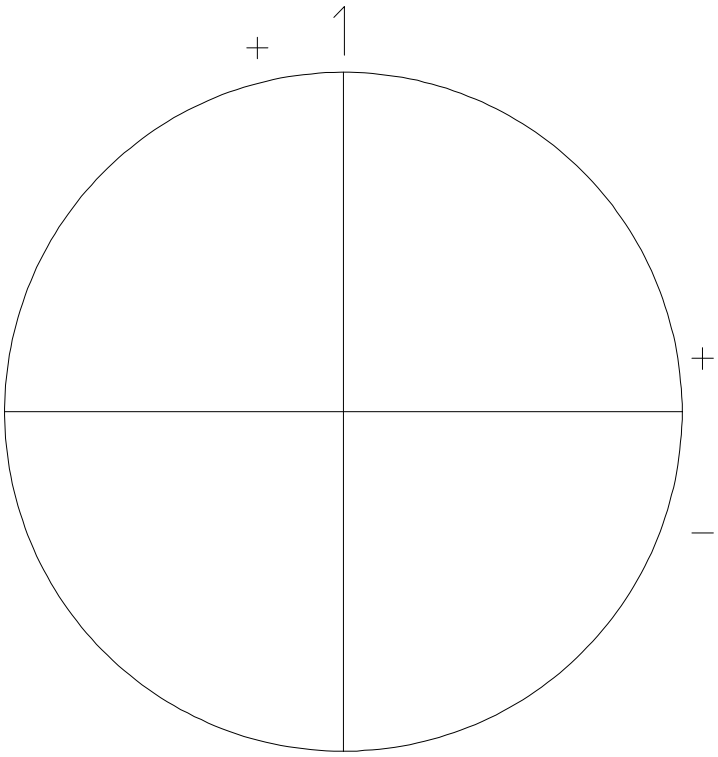
$$7 + 1 - 1 = 7$$

$$7 + 1 - 1 - (1 \cdot 2) = 5$$

$$7 + 1 - 1 - (2 \cdot 2) = 3$$

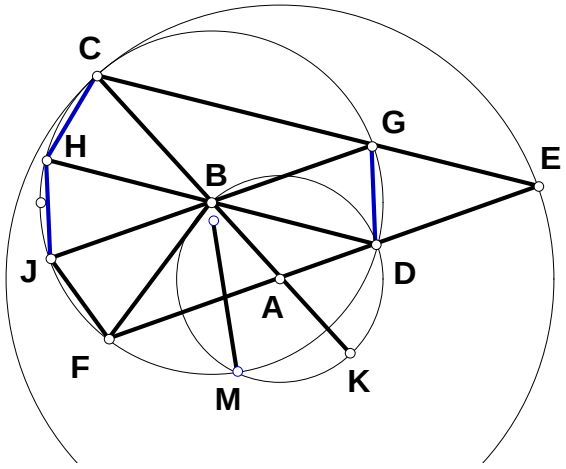
$$7 + 1 - 1 - 2 - 2 - 2 = 1$$

$$\mathrm{mod}(7 + 1 - 1, 2) = 1$$



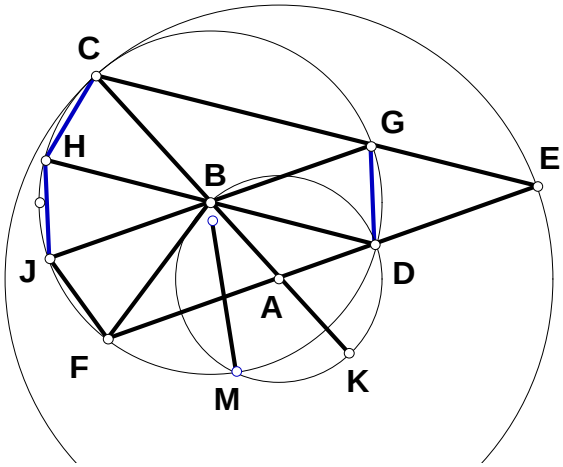
Work in progress.

010496 The Archamedian Paper Trisector- Without the Numbers.

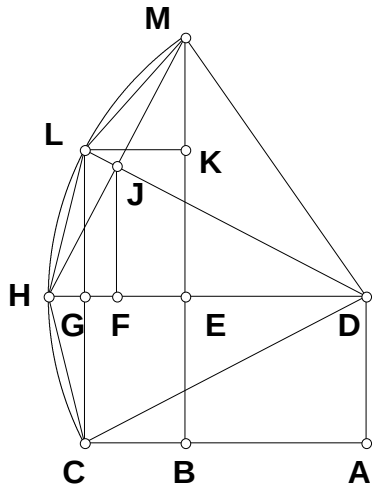


Given any circle AB.
Given any circle BC such that BC ≤ 2AB.
Construct AE such that AE = AC.
As AC = AB + BC
and AD = AB so too DE = BC.
Construct DH parallel to BD.
Construct CE.
As AB = AD and AC = AE,
Δ ABD is proportional to Δ ACE,
therefore CE is parallel to BD.
From here one can take two paths.
Construct GJ parallel to EF.
As CE is parallel to DH,
DG = CH.
As GJ is parallel to EF,
DG = FJ.

As ∠ HBJ is opposite
and equal to ∠ GBD,
DG = HJ,
therefore ∠ DG is $\frac{1}{3}$ CF.
As CE is parallel to DH
and EF is parallel to GJ,
EGBD is equilateral.
By construction DK = KM.
As DH is parallel to CE,
CH = DG.
As DK is equal and opposite CH,
MK + DK + DG is $\frac{1}{3}$ DG.



042897



$N := 1.458 \quad AD := 1 \quad AC := AD \cdot N \quad N := N$

$CD := \sqrt{AD^2 + AC^2} \quad DH := CD$

$CG := AD \quad DG := AC \quad GH := DH - DG \quad CH := \sqrt{GH^2 + CG^2}$

$HJ := CG \quad DJ := DG$

$FH := \frac{\left(HJ^2 + DH^2\right) - DJ^2}{2 \cdot DH} \quad EF := FH \quad DE := DH - (EF + FH)$

$AB := DE \quad EG := DG - DE \quad LM := CH \quad LK := EG$

$KM := \sqrt{LM^2 - LK^2} \quad BE := AD \quad BK := 2 \cdot BE \quad BM := BK + KM$

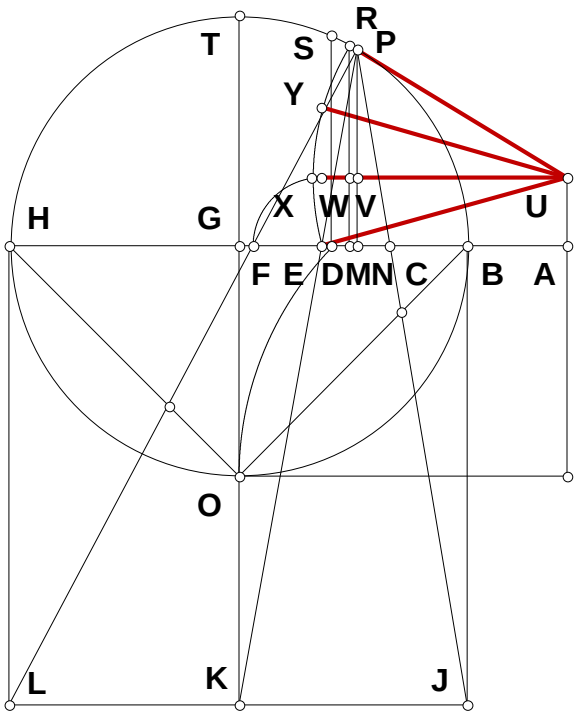
Some Algebraic Names:

$\sqrt{N^2 + 1} - CD = 0 \quad N - AC = 0 \quad \sqrt{N^2 + 1} - N - GH = 0 \quad \sqrt{2 \cdot N^2 - 2 \cdot \sqrt{N^2 + 1} \cdot N + 2} - CH = 0$

$\frac{1}{\sqrt{N^2 + 1}} - FH = 0 \quad \frac{(N - 1) \cdot (N + 1)}{\sqrt{N^2 + 1}} - AB = 0 \quad 2 + \sqrt{\frac{5 \cdot N^2}{N^2 + 1} - \frac{4 \cdot N}{\sqrt{N^2 + 1}} + \frac{1}{N^2 + 1}} - BM = 0$

Trisection and the Cube Roots 042997

If trisection can be placed at RUE, then PV is proportional to RW.



$N := 5 \qquad AB := 1 \qquad AH := AB \cdot N$

$BH := AH - AB \quad BJ := BH$

$AC := \left(AB^2 \cdot AH\right)^{\frac{1}{3}} \qquad AF := \left(AB \cdot AH^2\right)^{\frac{1}{3}}$

$CF := AF - AC \qquad CE := \frac{CF}{2}$

$AE := AC + CE$

$AU := CE \quad NV := AU \quad MW := AU$

(For the next two equations see 042897.)

$$AM := \frac{\left(\frac{AE}{AU} - 1\right) \cdot \left(\frac{AE}{AU} + 1\right) \cdot AU}{\sqrt{\left(\frac{AE}{AU}\right)^2 + 1}}$$

$$MR := 2 \cdot AU + AU \cdot \sqrt{\frac{5 \cdot \left(\frac{AE}{AU}\right)^2}{\left(\frac{AE}{AU}\right)^2 + 1} - \frac{4 \cdot \frac{AE}{AU}}{\sqrt{\left(\frac{AE}{AU}\right)^2 + 1}} + \frac{1}{\left(\frac{AE}{AU}\right)^2 + 1}} \qquad RW := MR - MW$$

$BC := AC - AB \qquad FH := AH - AF \qquad CN := \frac{BC \cdot CF}{BC + FH} \qquad NP := \frac{BJ \cdot CN}{BC} \qquad PV := NP - NV$

$AN := AC + CN \qquad UV := AN \quad UW := AM \qquad \frac{RW \cdot UV}{UW} - PV = 0$

m∠OKL = 30.872°

m∠HAQ = 20.581°

m∠OTK = 15.436°

m∠IJV = 10.291°

m∠IBJ = 5.145°

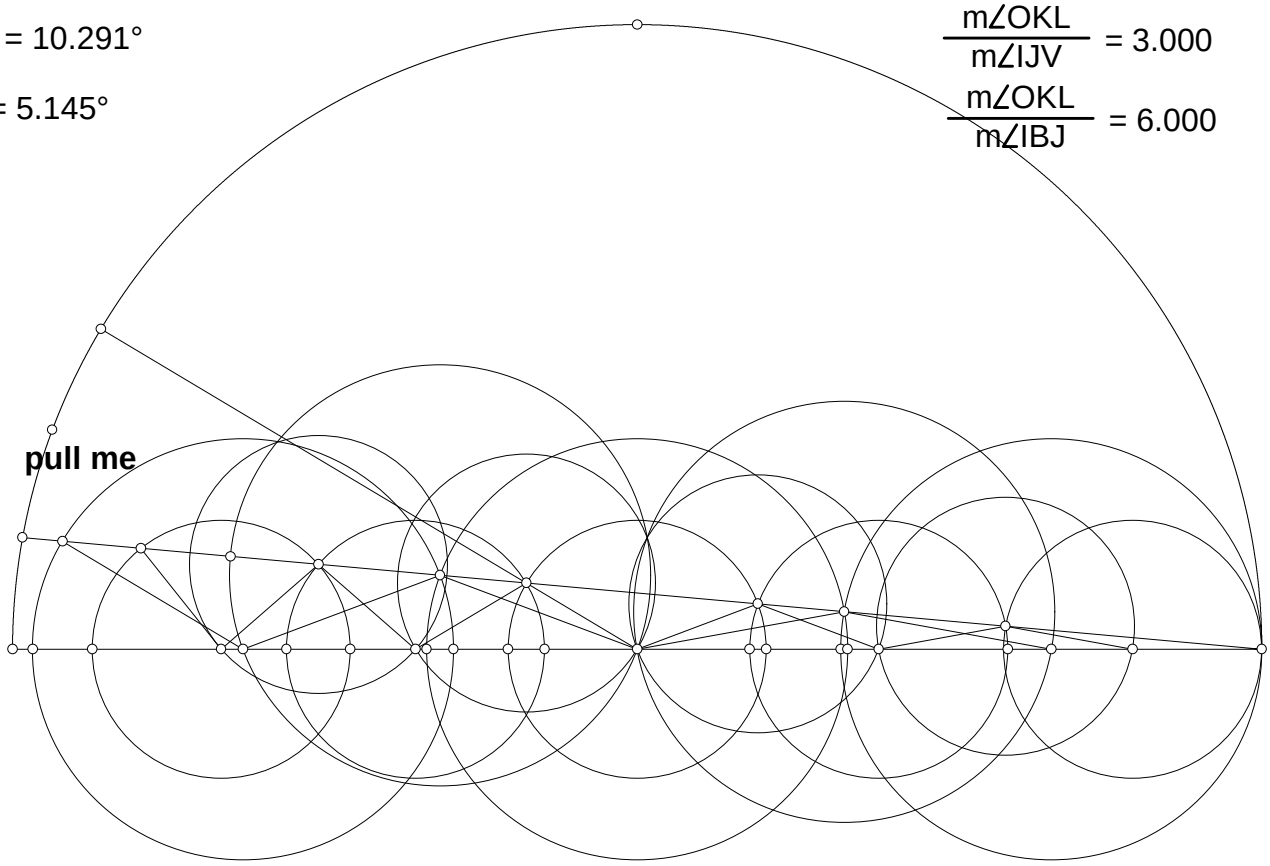
▲ **Show** △ **Hide**

$\frac{m\angle OKL}{m\angle HAQ} = 1.500$

$\frac{m\angle OKL}{m\angle OTK} = 2.000$

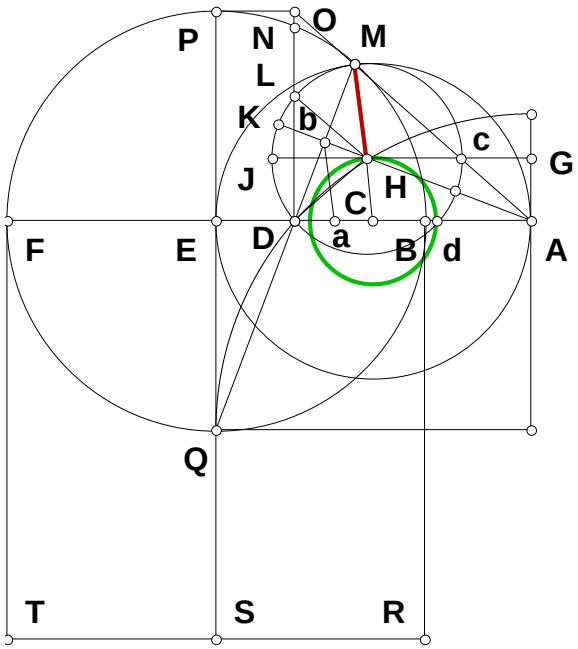
$\frac{m\angle OKL}{m\angle IJV} = 3.000$

$\frac{m\angle OKL}{m\angle IBJ} = 6.000$



A Square Root Figure And Triseciton 042398

In this square root figure, what is the radius of the circle with the trisected angle? What is the radius CH?



$N := 5 \quad AB := 1 \quad AF := AB \cdot N \quad BF := AF - AB$

$AD := (AB \cdot AF)^{\frac{1}{2}} \quad BE := \frac{BF}{2} \quad BD := AD - AB$

$DE := BE - BD \quad EQ := BE \quad DQ := \left(DE^2 + EQ^2\right)^{\frac{1}{2}}$

$PQ := BF \quad QM := \frac{EQ \cdot PQ}{DQ} \quad DM := QM - DQ$

$AE := AB + BE \quad AC := \frac{AE}{2} \quad Db := \frac{DM}{2}$

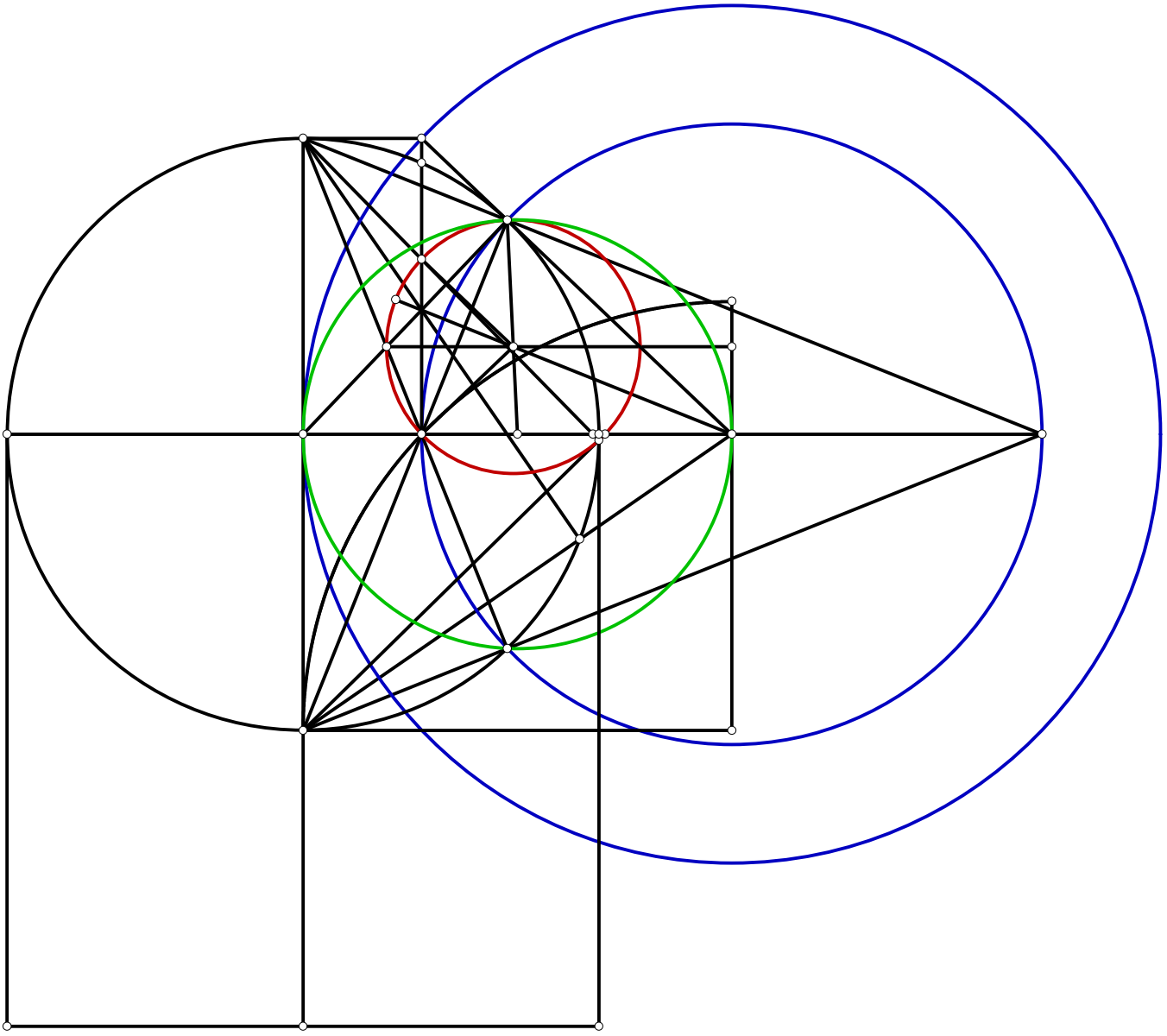
$CM := AC \quad ab := \frac{CM \cdot Db}{DM} \quad CD := AD - AC$

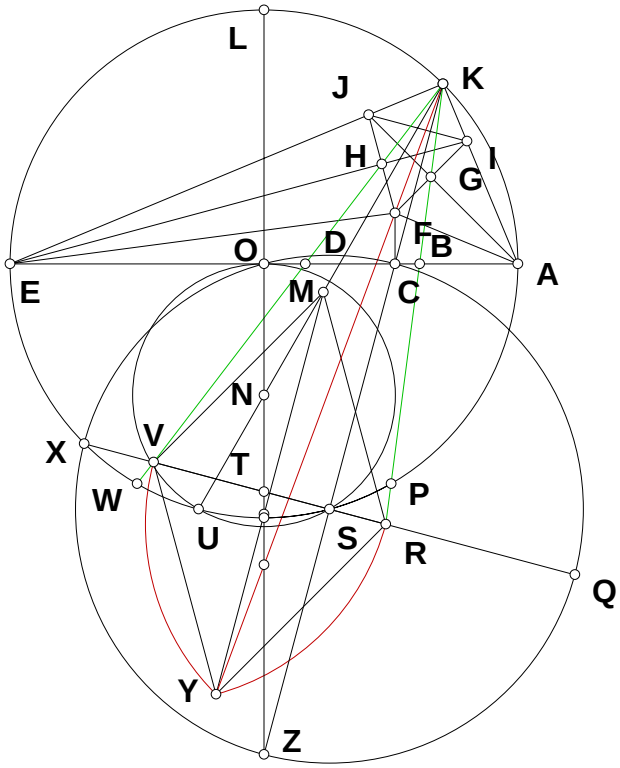
$Ca := \frac{CD}{2} \quad Aa := AC + Ca \quad CH := \frac{ab \cdot AC}{Aa}$

$AM := AD \quad Ac := \frac{AM \cdot CH}{CM} \quad HM := CM - CH$

$HM - Ac = 0$

$$HM - AB \cdot \frac{\sqrt{N} \cdot (N + 1)}{N + 4 \cdot \sqrt{N} + 1} = 0 \qquad CH - AB \cdot \frac{(N + 1)^2}{4 \cdot N + 16 \cdot \sqrt{N} + 4} = 0$$




L
K
J
H
I
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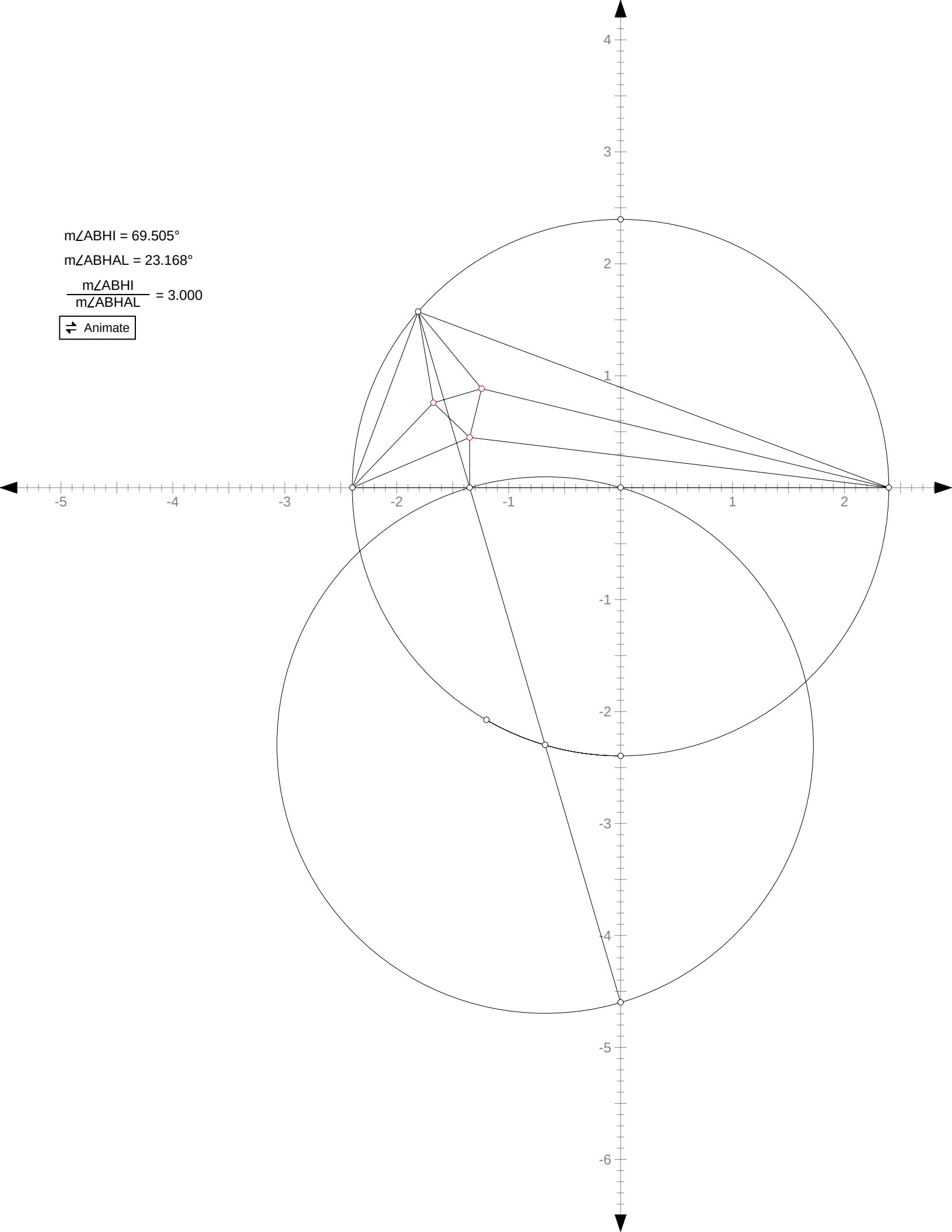
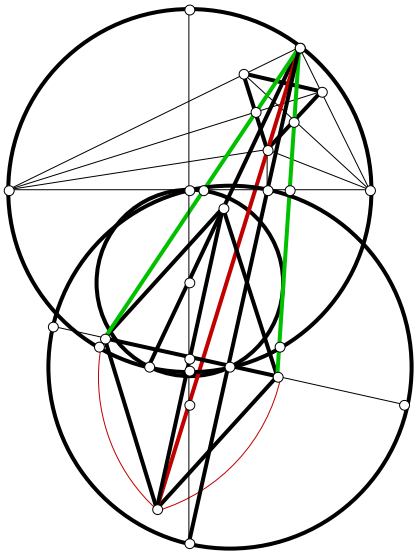

m∠ABHI = 69.505°
m∠ABHAL = 23.168°
m∠ABHI / m∠ABHAL = 3.000
Animate
4
3
2
1
-1
-2
-3
-4
-5
-6
-5
-4
-3
-2
-1
1
2

991002A

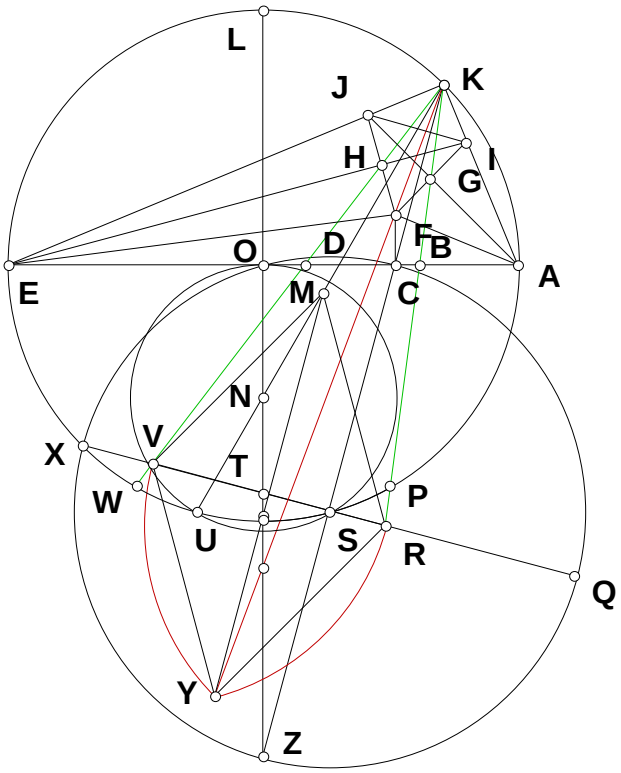


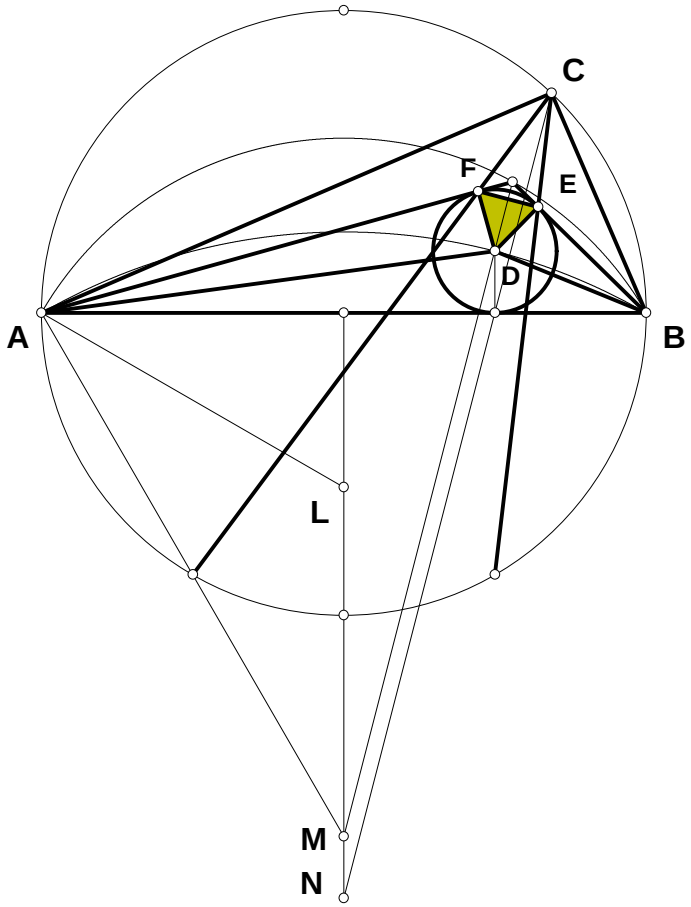
$N := 4.132 \quad AE := 2.6459 \quad CZ := AE$

$AO := \frac{AE}{2} \quad AC := \frac{AE}{N}$

$CO := AO - AC \quad OZ := \sqrt{CZ^2 - CO^2}$

$SZ := \frac{CZ}{2} \quad ST := \frac{CO \cdot SZ}{OZ}$




C
F
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M
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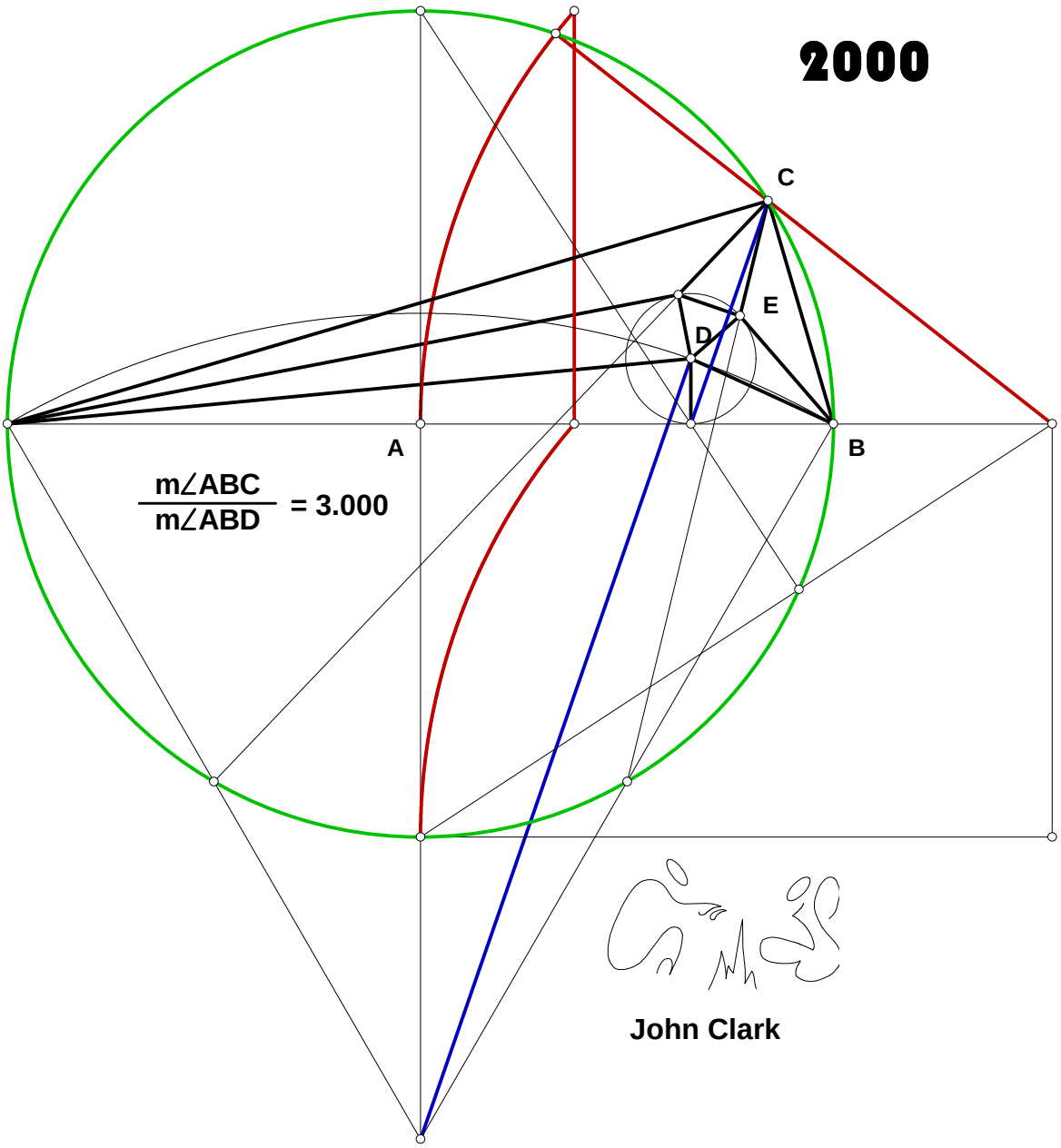
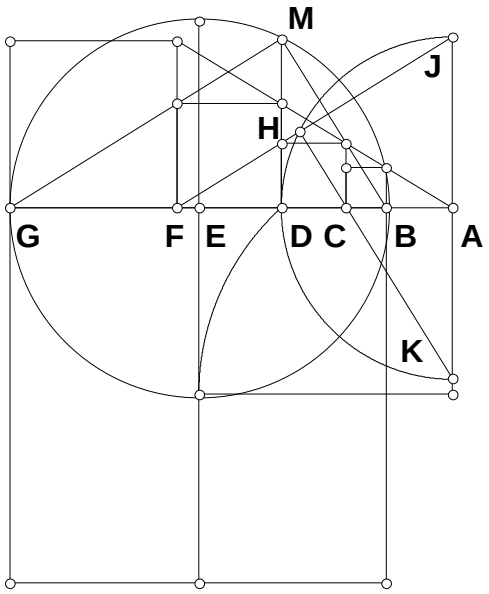
Three Pieces Of Paper



John Clark

07/09/00 Alternate Method Quad Roots



$N := 3 \quad AB := 1 \quad AG := AB \cdot N \quad AD := \sqrt{AB \cdot AG}$

$BD := AD - AB \quad BG := AG - AB$

$DG := BG - BD \quad DM := \sqrt{BD \cdot DG}$

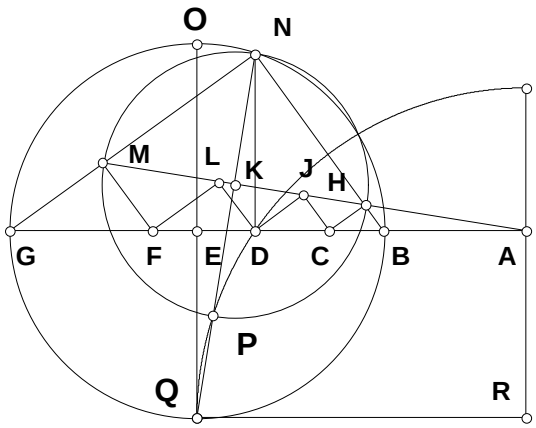
$AJ := AD \quad AK := AD \quad BM := \sqrt{BD^2 + DM^2}$

$GM := \sqrt{DG^2 + DM^2} \quad AF := \frac{GM \cdot AJ}{BM}$

$AC := \frac{BM \cdot AK}{GM}$

$\left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0 \quad \left(AB^3 \cdot AG\right)^{\frac{1}{4}} - AC = 0$

08/01/00 Alternate Method Quad Roots



$N := 5 \qquad AB := 1 \qquad AG := AB \cdot N$

$BG := AG - AB \qquad BE := \frac{BG}{2} \qquad AD := \sqrt{AB \cdot AG}$

$AE := AB + BE \qquad DE := AE - AD \qquad NY := DE$

$BD := AD - AB \qquad DG := BG - BD \qquad EQ := BE$

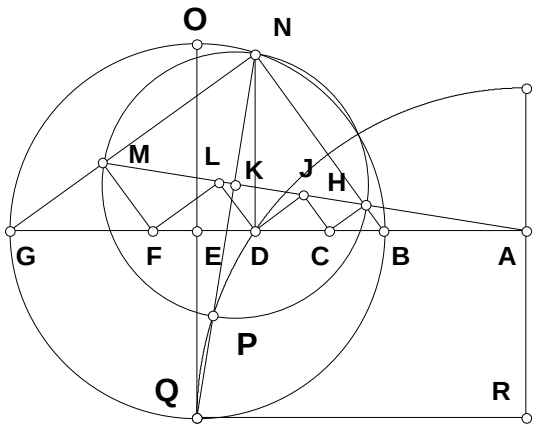
$DN := \sqrt{BD \cdot DG} \qquad NQ := \sqrt{DE^2 + (DN + EQ)^2}$

$QR := AE \qquad OQ := BG \qquad NO := \sqrt{OQ^2 - NQ^2}$

$PQ := \frac{NO \cdot 2 \cdot QR}{OQ} \qquad NP := NQ - PQ \qquad MN := \sqrt{\frac{NP^2}{2}} \qquad BN := \sqrt{BD^2 + DN^2} \qquad GN := \sqrt{DG^2 + DN^2}$

$GM := GN - MN \qquad FG := \frac{BG \cdot GM}{GN} \qquad AF := AG - FG \qquad \left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0$

08/01/00B Alternate Method Quad Roots



$N := 5 \qquad AB := 1 \qquad AG := AB \cdot N$

$BG := AG - AB \qquad BE := \frac{BG}{2} \qquad AD := \sqrt{AB \cdot AG}$

$BD := AD - AB \qquad DG := BG - BD \qquad EQ := BE$

$DN := \sqrt{BD \cdot DG} \qquad BN := \sqrt{BD^2 + DN^2}$

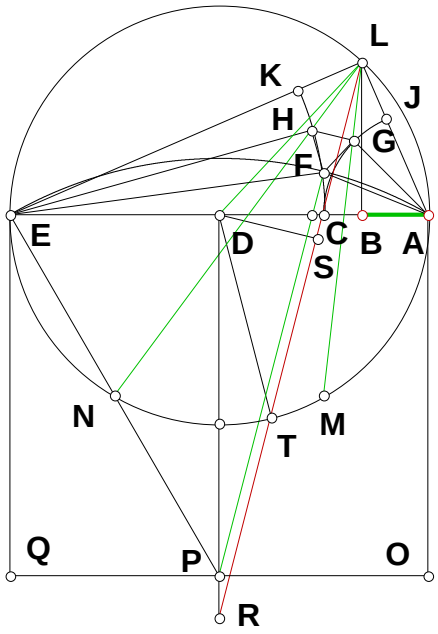
$GN := \sqrt{DG^2 + DN^2} \qquad DE := BE - BD$

$NQ := \sqrt{(DN + EQ)^2 + DE^2} \qquad AN := \sqrt{AD^2 + DN^2} \qquad AE := AB + BE \qquad AQ := \sqrt{AE^2 + EQ^2}$

$KN := \frac{NQ^2 + AN^2 - AQ^2}{2 \cdot NQ} \qquad KM := KN \qquad MN := \sqrt{KN^2 + KM^2} \qquad GM := GN - MN \qquad GF := \frac{BG \cdot GM}{GN}$

$AF := AG - GF \qquad \left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0$

In Trisection What Is AB? 08/02/00



In the trisection figure given and given AC as the Unit what is AB?

$AC := .884 \quad AE := 3.521 \quad AD := \frac{AE}{2}$

$EP := AE \quad DE := AD \quad DP := \sqrt{EP^2 - DE^2} \quad FP := EP$

$CE := AE - AC \quad CD := CE - DE \quad CF := \sqrt{FP^2 - CD^2} - DP$

$PR := CF \quad DR := DP + PR \quad CR := \sqrt{CD^2 + DR^2}$

$CS := \frac{CD^2}{CR} \quad DS := \sqrt{CD^2 - CS^2} \quad DL := AD$

$LS := \sqrt{DL^2 - DS^2} \quad RS := CR - CS \quad LR := RS + LS$

$BD := \frac{CD \cdot LR}{CR} \quad AB := AD - BD \quad ST := LS \quad RT := RS - ST$

In trisection the length RT to the similarity point is equal to the radius of the circle.

$$RT - \left(\frac{1}{2}\right) \cdot AE = 0$$

$$AD - \frac{AE}{2} = 0 \qquad DP - \frac{AE}{2} \cdot \sqrt{3} = 0 \qquad CF - \left[\frac{1}{2} \cdot \sqrt{-(AE + 2 \cdot AC) \cdot (-3 \cdot AE + 2 \cdot AC)} - \frac{1}{2} \cdot AE \cdot \sqrt{3}\right] = 0$$

$$CE - (AE - AC) = 0 \qquad CD - \left(\frac{1}{2} \cdot AE - AC\right) = 0 \qquad DR - \frac{1}{2} \cdot \sqrt{(AE + 2 \cdot AC) \cdot (3 \cdot AE - 2 \cdot AC)} = 0$$

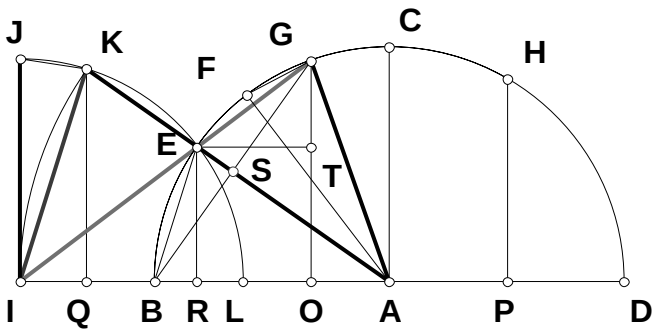
$$CR - AE = 0 \qquad CS - \left(\frac{1}{4}\right) \cdot \frac{(-AE + 2 \cdot AC)^2}{AE} = 0 \qquad LS - \frac{1}{4} \cdot \frac{\left(-4 \cdot AC^2 + 4 \cdot AE \cdot AC + AE^2\right)}{AE} = 0$$

$$DS - \frac{1}{4} \cdot \frac{(AE - 2 \cdot AC)}{AE} \cdot \sqrt{(AE + 2 \cdot AC) \cdot (3 \cdot AE - 2 \cdot AC)} = 0 \qquad LR - \frac{\left(AE^2 - 2 \cdot AC^2 + 2 \cdot AE \cdot AC\right)}{AE} = 0$$

$$RS - \frac{1}{4} \cdot (AE + 2 \cdot AC) \cdot \frac{(3 \cdot AE - 2 \cdot AC)}{AE} = 0 \qquad BD - \left(\frac{1}{2} \cdot AE - \frac{3}{AE} \cdot AC^2 + \frac{2}{AE^2} \cdot AC^3\right) = 0$$

$$AB - AC^2 \cdot \frac{(3 \cdot AE - 2 \cdot AC)}{AE^2} = 0 \qquad AB \cdot AE^2 - AC^2 \, (3 \cdot AE - 2 \cdot AC) = 0$$

080300 Trisection



If 2 IQ = EK then 2 JK = EK and the figure projected from BCD will yield a trisected figure JKL.

$N := 3 \qquad BD := 2$

$AB := \frac{BD}{2} \quad AD := AB \quad AP := \frac{AD}{2}$

$BP := AB + AP \quad BO := \frac{BP}{N} \quad AE := AB$

$DO := BD - BO \quad GO := \sqrt{BO \cdot DO}$

$BG := \sqrt{GO^2 + BO^2} \quad BS := \frac{BG}{2} \quad ER := BS \quad TO := ER \quad GT := GO - TO \quad AS := \sqrt{AB^2 - BS^2}$

$ES := AE - AS \quad BR := ES \quad OR := BO - BR \quad ET := OR \quad IO := \frac{ET \cdot GO}{GT} \quad BI := IO - BO$

$AI := BI + AB \quad BE := \sqrt{ER^2 + BR^2} \quad GE := BE \quad GI := \frac{GE \cdot GO}{GT} \quad EI := GI - GE \quad AK := AI \quad IK := EI$

$IQ := \frac{IK^2 + AI^2 - AK^2}{2 \cdot AI} \quad EK := AK - AE \quad \frac{EK}{IQ} = 2$

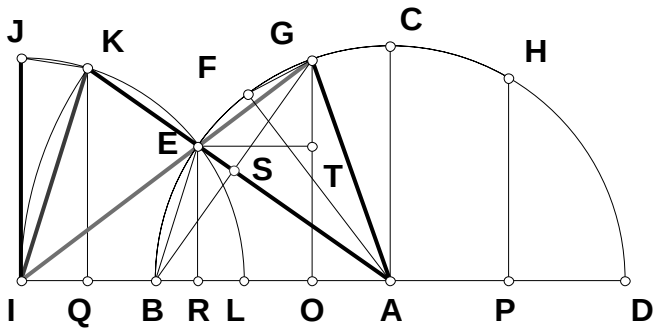
Some Algebraic Names:

$$\frac{3 \cdot BD}{4 \cdot N} - BO = 0 \quad \frac{BD}{4} \cdot \frac{(4 \cdot N - 3)}{N} - DO = 0 \quad \frac{BD}{(4 \cdot N)} \cdot \sqrt{3} \cdot \sqrt{4 \cdot N - 3} - GO = 0 \quad \frac{BD}{2} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N}} - BG = 0$$

$$\frac{BD}{4} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N}} - BS = 0 \quad \frac{BD}{4} \cdot \sqrt{3} \cdot \frac{\left(\sqrt{4 \cdot N - 3} - \sqrt{\frac{1}{N}} \cdot N\right)}{N} - GT = 0 \quad \frac{BD}{4} \cdot \sqrt{\frac{(4 \cdot N - 3)}{N}} - AS = 0$$

$$\frac{BD}{4} \cdot \left[2 - \sqrt{\frac{(4 \cdot N - 3)}{N}}\right] - ES = 0 \quad \frac{BD}{4} \cdot \frac{\left[3 - 2 \cdot N + \sqrt{\frac{(4 \cdot N - 3)}{N}} \cdot N\right]}{N} - OR = 0$$

$$\frac{-BD}{4} \cdot \frac{\left[3 \cdot \sqrt{4 \cdot N - 3} \cdot \sqrt{N} - 2 \cdot \sqrt{4 \cdot N - 3} \cdot N^{\left(\frac{3}{2}\right)} + 4 \cdot N^2 - 3 \cdot N\right]}{\left[N^{\left(\frac{3}{2}\right)} \cdot \left(-\sqrt{4 \cdot N - 3} + \sqrt{N}\right)\right]} - IO = 0 \quad \frac{-BD}{2} \cdot \frac{\left(\sqrt{4 \cdot N - 3} - 2 \cdot \sqrt{N}\right)}{\left(\sqrt{4 \cdot N - 3} - \sqrt{N}\right)} - BI = 0$$



$$\frac{-BD}{\left(-2\cdot\sqrt{4\cdot N-3}+2\cdot\sqrt{N}\right)}\cdot\sqrt{N}-AI=0$$

$$\frac{BD}{2}\cdot\sqrt{2-\frac{1}{\sqrt{N}}\cdot\sqrt{4\cdot N-3}}-BE=0$$

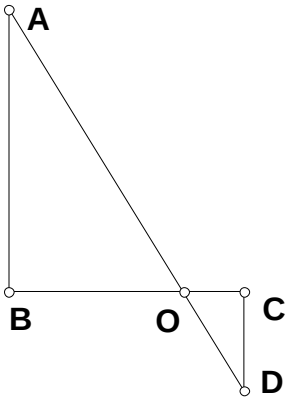
$$\frac{BD}{2}\cdot\sqrt{\frac{-\left(-2\cdot\sqrt{N}+\sqrt{4\cdot N-3}\right)}{\sqrt{N}}}\cdot\frac{\sqrt{4\cdot N-3}}{\left(\sqrt{4\cdot N-3}-\sqrt{N}\right)}-GI=0$$

$$\frac{-BD}{2}\cdot\sqrt{2-\frac{\sqrt{4\cdot N-3}}{\sqrt{N}}}\cdot\frac{\sqrt{N}}{\left(-\sqrt{4\cdot N-3}+\sqrt{N}\right)}-EI=0$$

$$\frac{BD}{4}\cdot\frac{\left(2\cdot\sqrt{N}-\sqrt{4\cdot N-3}\right)}{\left(\sqrt{4\cdot N-3}-\sqrt{N}\right)}-IQ=0\qquad\frac{BD}{2}\cdot\frac{\left(2\cdot\sqrt{N}-\sqrt{4\cdot N-3}\right)}{\left(\sqrt{4\cdot N-3}-\sqrt{N}\right)}-EK=0$$

08/07/00 Proportion Series II

Divide BC into the same ratio as AB:CD.



$AB := 9 \quad CD := 3 \quad BC := 5$

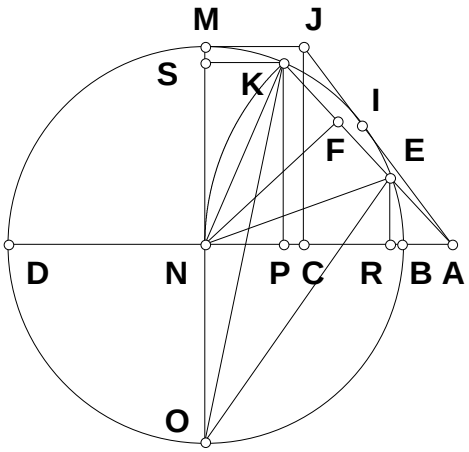
$BO := \frac{AB \cdot BC}{AB + CD} \quad CO := \frac{CD \cdot BC}{AB + CD}$

$BO = 3.75 \qquad CO = 1.25$

$BO + CO - BC = 0$

$\frac{AB}{CD} - \frac{BO}{CO} = 0$

08/23/00 Trisection In A Square Root Figure



Given the square root figure drawn for trisection, what is AR given AB and AD?

$N := 4 \qquad AB := 2 \qquad AD := AB \cdot N$

$AC := \sqrt{AB \cdot AD} \qquad BD := AD - AB$

$BN := \frac{BD}{2} \qquad KN := BN$

$AN := AB + BN \qquad AK := AN$

$AP := \frac{AK^2 + AN^2 - KN^2}{2 \cdot AN} \qquad AF := \frac{AP \cdot AN}{AK}$

$FK := AK - AF \qquad EF := FK$

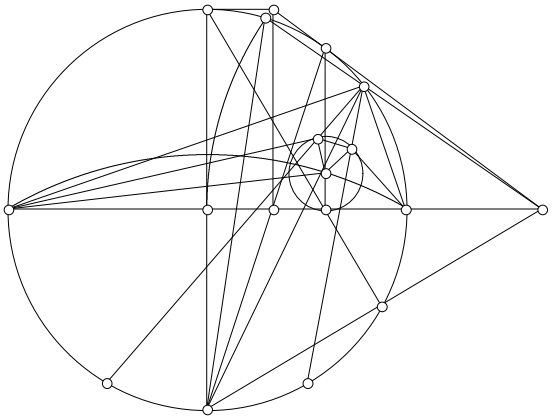
$AE := AK - 2 \cdot (EF) \qquad AR := \frac{AP \cdot AE}{AK} \qquad AB \cdot N \cdot \frac{\left(N^2 + 6 \cdot N + 1\right)}{(N+1)^3} - AR = 0 \qquad BR := AR - AB$

$AB \cdot (3 \cdot N + 1) \cdot \frac{(N-1)}{(N+1)^3} - BR = 0$

Does KS = FK?

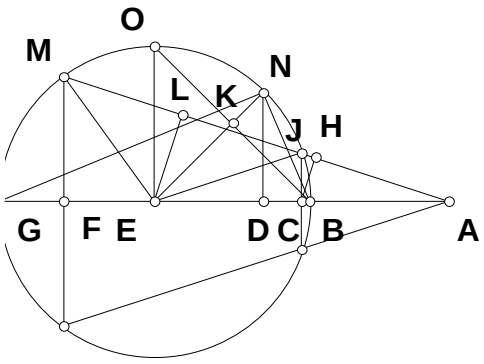
$BP := AP - AB \qquad DP := BD - BP \qquad NP := BN - BP \qquad KS := NP \qquad KS - FK = 0$

$\frac{AB \cdot (N-1)^2}{4 \cdot (N+1)} - KS = 0$



09/03/00 Ratios In Trisection

How does BF vary with BC? How does DF vary with BC?



$N_1 := 4 \quad N_2 := 8$

$BG := 1 \quad BE := \frac{BG}{2} \quad EM := BE \quad BO := \sqrt{2 \cdot BE^2}$

$EN := BE \quad EK := \frac{BE \cdot BE}{BO} \quad KN := EN - EK \quad BK := \frac{BO}{2}$

$BN := \sqrt{BK^2 + KN^2} \quad BD := \frac{BN^2}{BG} \quad BC := BD \cdot \frac{N_1}{N_2}$

$CG := BG - BC \quad CJ := \sqrt{BC \cdot CG} \quad AJ := BE$

$AC := \sqrt{AJ^2 - CJ^2} \quad AB := AC - BC \quad AE := AB + BE$

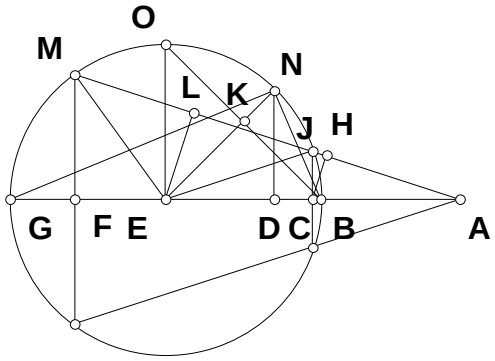
$JH := \frac{CJ^2}{AJ} \quad AH := AJ - JH \quad AL := \frac{AH \cdot AE}{AC} \quad JL := AL - AJ$

$LM := JL \quad AM := AL + LM \quad AF := \frac{AH \cdot AM}{AC} \quad BF := AF - AB$

$$\frac{-1}{4} \cdot \left(-2 + \sqrt{2}\right) \cdot \frac{N_1}{N_2} - BC = 0$$

$$BF - \frac{1}{8} \cdot \left(7 \cdot \sqrt{2} - 10\right) \cdot \left(N_1 - 4 \cdot N_2 - 2 \cdot N_2 \cdot \sqrt{2}\right) \cdot \frac{\left(2 \cdot N_1 - 2 \cdot N_2 - N_2 \cdot \sqrt{2}\right)^2}{N_2^3} = 0$$

$$\frac{1}{2} \cdot \frac{\left(2 \cdot N_1 - 2 \cdot N_2 - N_2 \cdot \sqrt{2}\right)^2 \cdot \left[\left(2 \cdot \sqrt{2} - 3\right) \cdot \left(N_1 - 4 \cdot N_2 - 2 \cdot N_2 \cdot \sqrt{2}\right)\right]}{\left(N_2^2 \cdot N_1\right)} - \frac{BF}{BC} = 0$$

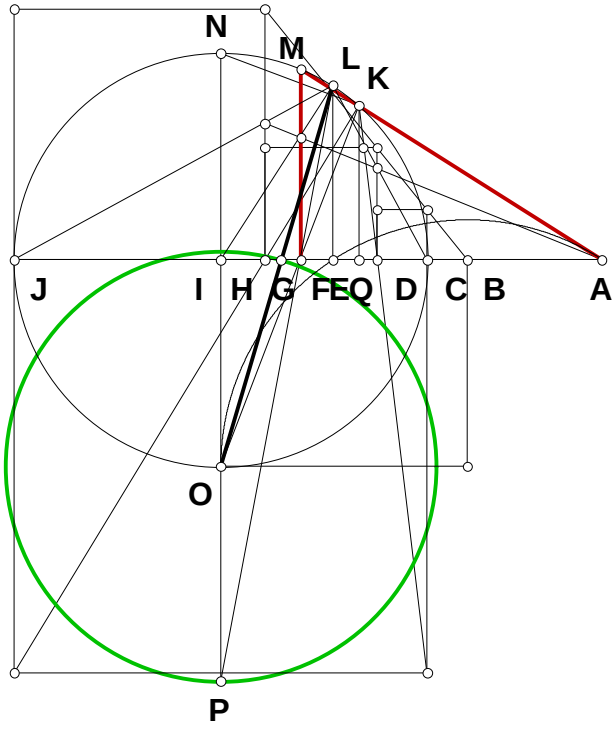


$$DF := BF - BD$$

$$\frac{\left(N_1 - N_2\right)\cdot\left[\left(7\cdot\sqrt{2} - 10\right)\cdot\left(12\cdot\sqrt{2}\cdot N_2^{\,2} + 17\cdot N_2^{\,2} + 2\cdot N_1^{\,2} - 10\cdot N_1\cdot N_2 - 6\cdot N_1\cdot\sqrt{2}\cdot N_2\right)\right]}{4\,N_2^{\,3}} - DF = 0$$

$$\frac{\left(N_1 - N_2\right)\cdot\left[\left(2\cdot\sqrt{2} - 3\right)\cdot\left(2\cdot N_1^{\,2} - 10\cdot N_1\cdot N_2 - 6\cdot N_1\cdot\sqrt{2}\cdot N_2 + 17\cdot N_2^{\,2} + 12\cdot\sqrt{2}\cdot N_2^{\,2}\right)\right]}{N_1\cdot N_2^{\,2}} - \frac{DF}{BC} = 0$$

Goshdarn Good Pencil 09/16/00



$N := 2$

$BC := 1 \quad BJ := BC \cdot N \quad BE := \sqrt{BC \cdot BJ}$

$CJ := BJ - BC \quad CI := \frac{CJ}{2} \quad IO := CI$

$NO := CJ \quad CR := CJ$

$CE := BE - BC \quad EI := CI - CE \quad EJ := CJ - CE$

$EL := \sqrt{CE \cdot EJ} \quad EG := \frac{EI \cdot EL}{EL + IO} \quad GI := EI - EG$

$GO := \sqrt{GI^2 + IO^2} \quad OP := GO$

$IP := IO + OP \quad EF := \frac{EI \cdot EL}{EL + IP} \quad FI := EI - EF$

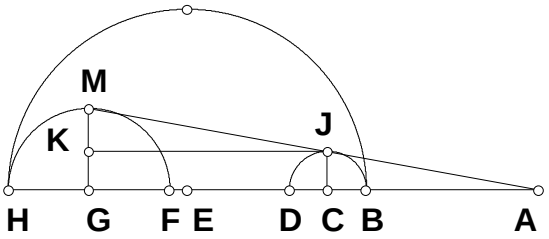
$FO := \sqrt{FI^2 + IO^2} \quad OK := \frac{IO \cdot NO}{FO}$

$FK := OK - FO \quad FQ := \frac{FI \cdot FK}{FO} \quad QI := FQ + FI \quad CQ := CI - QI \quad QJ := CJ - CQ \quad QK := \sqrt{CQ \cdot QJ}$

$CD := \frac{CQ \cdot CR}{CR + QK} \quad BD := CD + BC \quad \left(BC^2 \cdot BJ\right)^{\frac{1}{3}} - BD = 0.000004486957912$

Midpoints and Similarity Points 09/18/00

What is AE given the radius of the two circles and the difference between their centers? (External Unit).



$N_1 := 2 \quad N_2 := 3 \quad N_3 := 8$

$BC := N_1 \quad GH := N_2 \quad CG := N_3$

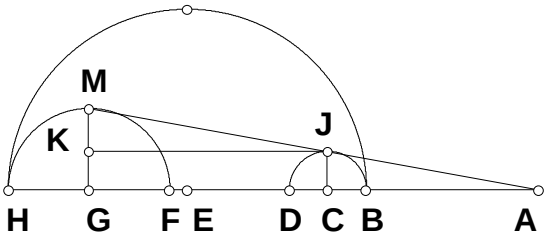
$CJ := BC \quad GM := GH \quad GK := CJ$

$BH := BC + CG + GH \quad BE := \frac{BH}{2}$

$KM := GM - GK \quad JK := CG \quad AG := \frac{JK \cdot GM}{KM} \quad AH := AG + GH \quad AB := AH - BH \quad AE := AB + BE$

$$AE - \frac{N_3 \cdot N_2 - 2 \cdot N_1 \cdot N_2 + N_1^2 + N_3 \cdot N_1 + N_2^2}{2 \cdot (N_2 - N_1)} = 0$$

What is AE if given the difference between their extremes and the radius given as a ratio of that segment? (Internal Unit).



$N_1 := 3 \quad N_2 := 7 \quad N_3 := 4 \quad N_4 := 9$

$BH := 1 \quad BE := \frac{BH}{2} \quad BC := BH \cdot \frac{N_1}{N_2}$

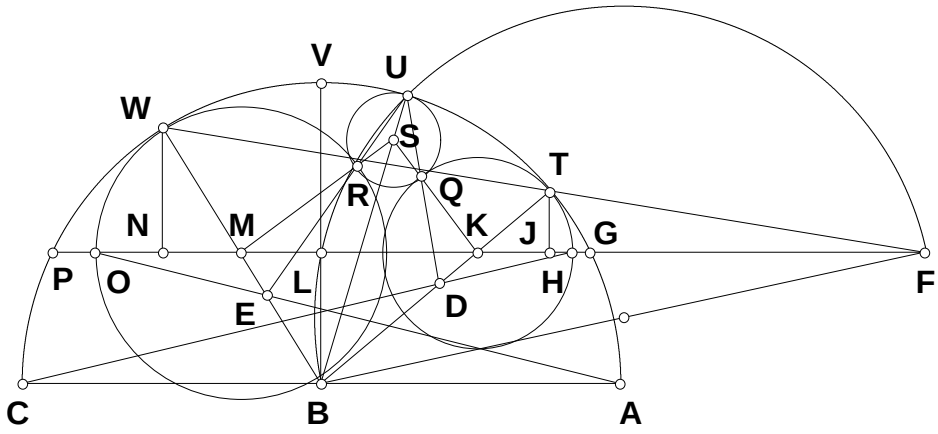
$GH := BH \cdot \frac{N_3}{N_4} \quad CG := BH - (BC + GH)$

$CJ := BC \quad GM := GH \quad GK := CJ$

$KM := GM - GK \quad JK := CG \quad AG := \frac{JK \cdot GM}{KM} \quad AH := AG + GH \quad AB := AH - BH \quad AE := AB + BE$

$$AE - \frac{(N_3 \cdot N_2 - 4 \cdot N_3 \cdot N_1 + N_1 \cdot N_4)}{2 \cdot (N_3 \cdot N_2 - N_1 \cdot N_4)} = 0$$

100600



$N_1 := 2.286$

$N_2 := 4.805$

$N_3 := 1.5364$

$AC := 3.104$

$AB := \frac{AC}{2}$ $BW := AB$

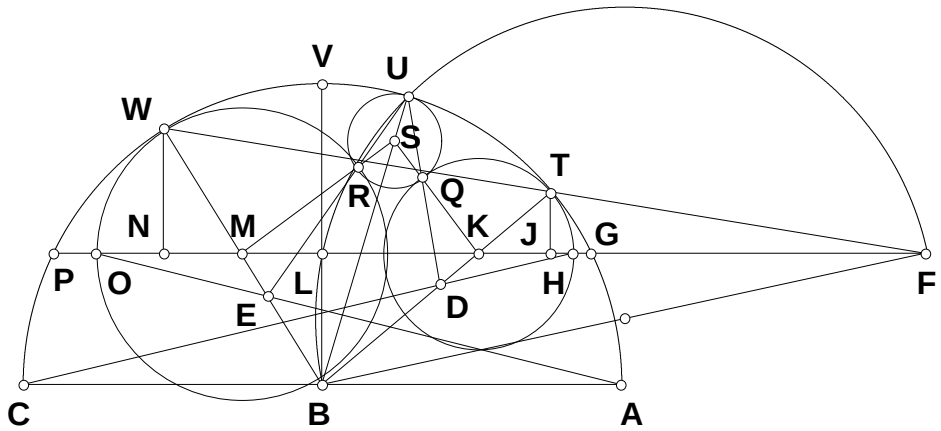
$BT := AB$ $BV := AB$

$BL := \frac{BV}{N_1}$ $LV := BV - BL$ $LP := (LV \cdot (BL + BV))^{\frac{1}{2}}$ $GL := LP$ $GP := GL + LP$ $GK := \frac{GP}{N_2}$

$GM := \frac{GP}{N_3}$ $KL := GL - GK$ $LM := GM - GL$ $BK := \left(BL^2 + KL^2\right)^{\frac{1}{2}}$ $BM := \left(BL^2 + LM^2\right)^{\frac{1}{2}}$

$KT := BT - BK$ $MW := BW - BM$ $KM := GM - GK$ $D := KM$ $R_L := MW$ $R_S := KT$

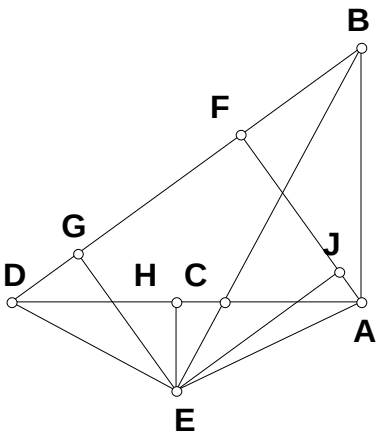
$FM := \frac{D \cdot R_L}{R_L - R_S}$ $FL := FM - LM$ $BF := \left(BL^2 + FL^2\right)^{\frac{1}{2}}$



11/13/00

Given AB, DE, AD find BE, AC, CD, CE, BC.

BAD and BED are right.



$N_1 := 3 \qquad N_2 := 5 \qquad N_3 := 1$

$AB := N_1 \quad AD := N_2 \quad DE := N_3$

$BD := \sqrt{AB^2 + AD^2} \quad BF := \frac{AB^2}{BD} \quad DG := \frac{DE^2}{BD} \qquad BE := \sqrt{BD^2 - DE^2}$

$AF := \sqrt{AB^2 - BF^2} \quad EG := \sqrt{DE^2 - DG^2} \qquad FG := BD - (BF + DG)$

$EJ := FG \quad FJ := EG \quad AJ := AF - FJ \qquad AE := \sqrt{EJ^2 + AJ^2}$

$S_1 := AD \quad S_2 := DE \quad S_3 := AE \qquad AH := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1}$

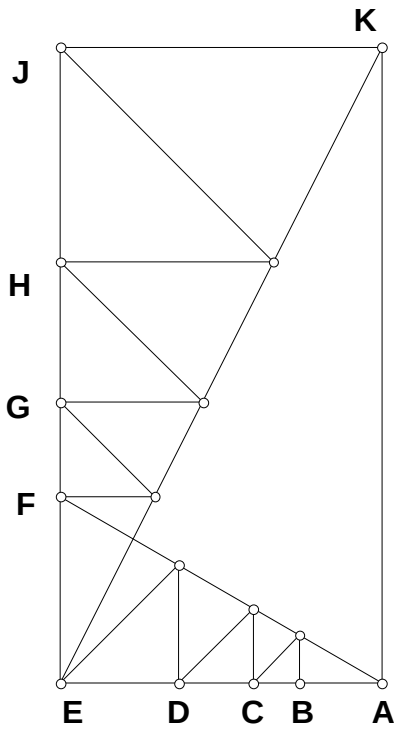
$EH := \sqrt{AE^2 - AH^2} \quad CH := \frac{EH \cdot AH}{AB + EH} \quad AC := AH - CH \quad CE := \frac{AC \cdot DE}{AB} \quad CD := AD - AC \quad BC := BE - CE$

$$N_1 \cdot \frac{\left(-N_3^2 \cdot N_2 + N_2^3 + N_1^2 \cdot N_2 - N_1 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2} \cdot N_3\right)}{\left(N_1^3 + N_1 \cdot N_2^2 + N_3 \cdot \sqrt{-N_3^2 \cdot N_2^2 + N_2^4 + N_1^2 \cdot N_2^2 - 2 \cdot N_1 \cdot N_2 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2} \cdot N_3 + N_3^2 \cdot N_1^2}\right)} - AC = 0$$

$$N_3 \cdot \frac{\left(-N_3^2 \cdot N_2 + N_2^3 + N_2 \cdot N_1^2 - N_1 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}\right)}{\left(N_1^3 + N_1 \cdot N_2^2 + N_3 \cdot \sqrt{-N_3^2 \cdot N_2^2 + N_2^4 + N_1^2 \cdot N_2^2 - 2 \cdot N_1 \cdot N_2 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2} \cdot N_3 + N_3^2 \cdot N_1^2}\right)} - CE = 0$$

Means On Means 11/28/00

Modify 02/28/98 for Mean proportionals between E and J.



$AE := 1 \qquad N := 3 \qquad EJ := AE \cdot N \qquad JK := AE$

$HJ := \frac{JK \cdot EJ}{JK + EJ} \qquad EH := EJ - HJ \qquad GH := \frac{EH \cdot HJ}{EH + HJ}$

$EG := EH - GH \qquad FG := \frac{EG \cdot GH}{EG + GH} \qquad EF := EG - FG$

$DE := \frac{EF \cdot AE}{EF + AE} \qquad AD := AE - DE \qquad CD := \frac{AD \cdot DE}{AD + DE}$

$AC := AD - CD \qquad BC := \frac{AC \cdot CD}{AC + CD} \qquad AB := AC - BC$

$M := 0 .. 3 \qquad P := 0 .. 3 \qquad AEAB_{M,P} := \left[\frac{N^{M+1}}{(N+1)^{M}} + 1 \right]^{P}$

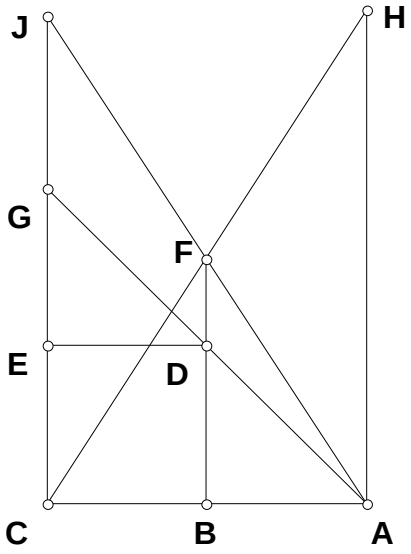
$$AEAB = \begin{bmatrix} 1 & 4 & 16 & 64 \\ 1 & 3.25 & 10.563 & 34.328 \\ 1 & 2.688 & 7.223 & 19.411 \\ 1 & 2.266 & 5.133 & 11.63 \end{bmatrix}$$

$AEAB_{3,3} - \frac{AE}{AB} = 0 \qquad AEAB_{3,2} - \frac{AE}{AC} = 0 \qquad AEAB_{3,1} - \frac{AE}{AD} = 0 \qquad AEAB_{3,0} - \frac{AE}{AE} = 0$

$AEAB_{3,3} - \frac{AE}{AB} = 0$

Multiplication and Division-Line By A Line 11/29/00

Given some unit, and two differences, multiply or divide the one difference by the other.
For Division:



$AC := 1 \qquad N_1 := 3 \qquad N_2 := 12 \qquad AH := N_1$

$CJ := N_2 \qquad AB := \frac{AH}{(CJ + AH)} \cdot AC$

$BC := AC - AB \quad BD := BC \quad CG := \frac{BD \cdot AC}{AB}$

$CG - \frac{N_2}{N_1} = 0 \qquad CG = 4$

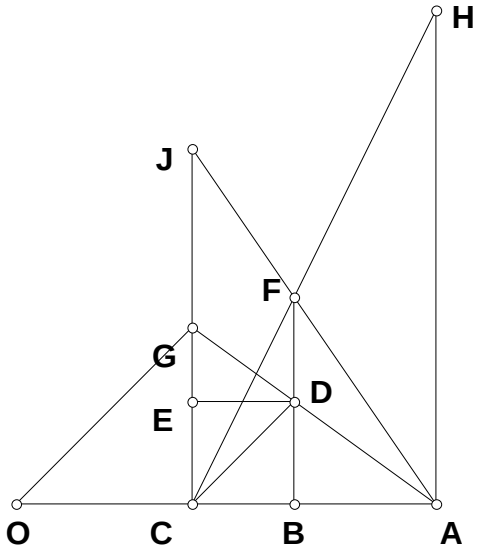
For Multiplication:

$AC := 1 \qquad N_1 := 5 \qquad N_2 := 7 \qquad AH := N_1$

$CG := N_2 \qquad CO := CG \qquad BD := \frac{CG \cdot AC}{AC + CO}$

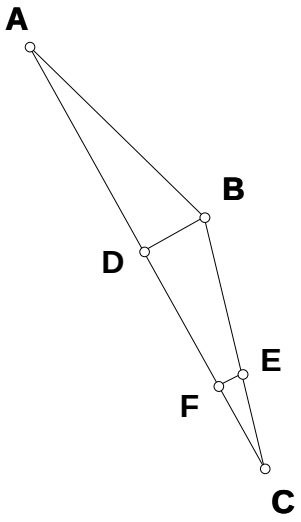
$BC := BD \qquad AB := AC - BC \qquad BF := \frac{AH \cdot BC}{AC}$

$CJ := BF \cdot \frac{AC}{AB} \qquad CJ - N_1 \cdot N_2 = 0 \qquad CJ = 35$



120500

From an observer C, the distance to star A and B are known, a reference CEF has been constructed, find the difference between the two stars.



$N_1 := 5 \qquad N_2 := 25 \qquad N_3 := 1 \qquad N_4 := .5$

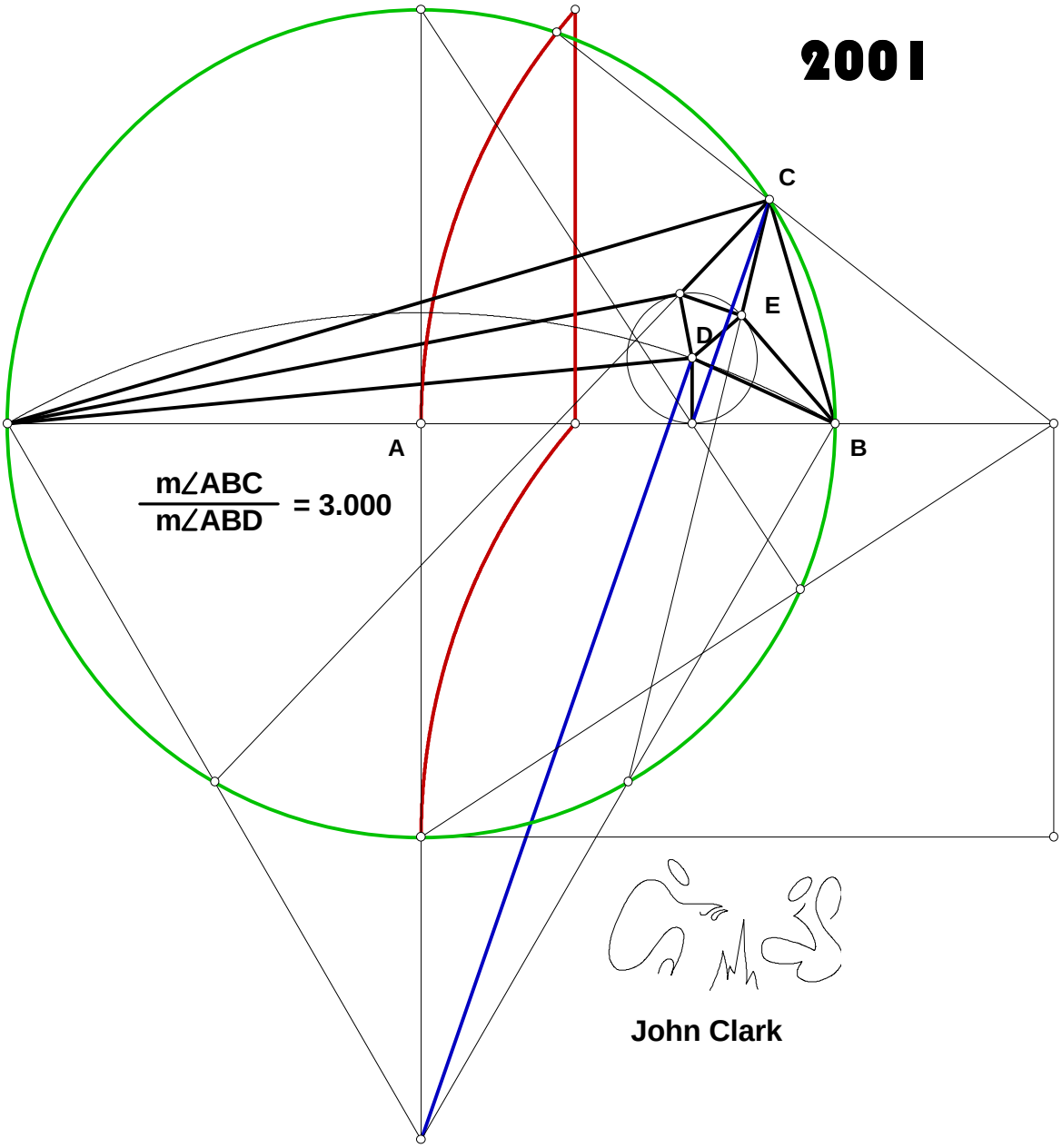
$BC := N_1 \qquad AC := N_2 \qquad CE := N_3 \qquad EF := N_4$

$BD := \frac{EF \cdot BC}{CE} \qquad CF := \sqrt{CE^2 - EF^2} \qquad CD := \frac{CF \cdot BC}{CE}$

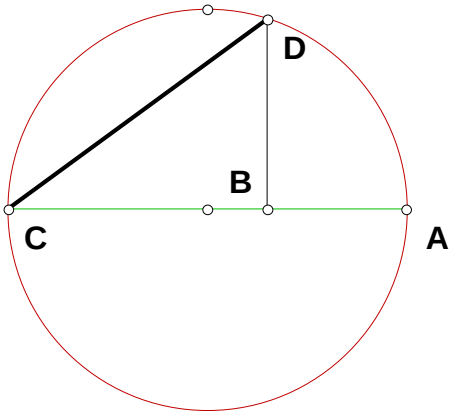
$AD := AC - CD \qquad AB := \sqrt{BD^2 + AD^2}$

$$AB - \sqrt{\frac{1}{N_3} \cdot \left(N_2^2 \cdot N_3 - 2 \cdot N_2 \cdot \sqrt{N_3^2 - N_4^2} \cdot N_1 + N_1^2 \cdot N_3 \right)} = 0$$

Three Pieces Of Paper



John Clark



Alternate method for common segment common endpoint square root. $\sqrt{AC \cdot BC} = CD$

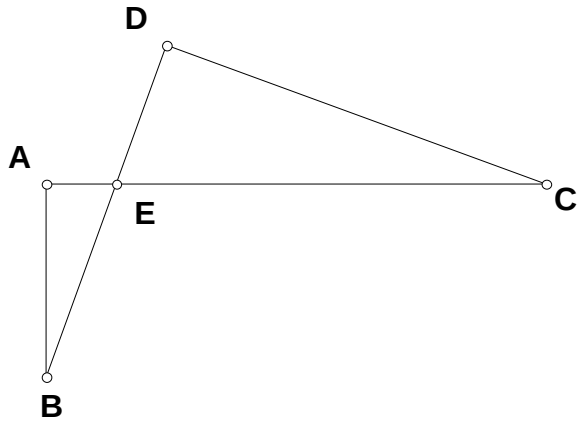
$N_1 := 5 \quad N_2 := 3 \quad AC := N_1 \quad BC := N_2$

$AB := AC - BC \quad BD := \sqrt{AB \cdot BC} \quad CD := \sqrt{BD^2 + BC^2}$

$\sqrt{N_2 \cdot N_1} - CD = 0$

Three Given Five Taken 042101

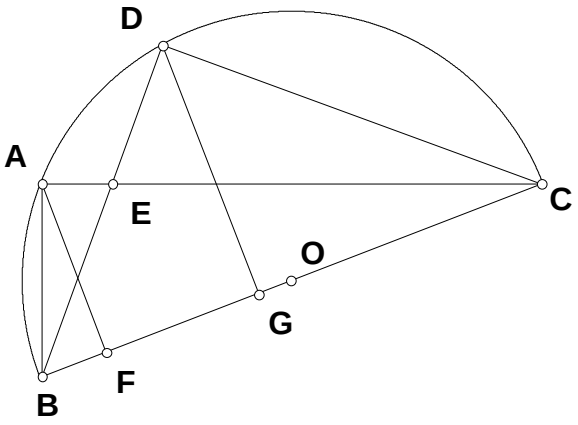
Given AB, CD, AC and that CDB, and BAC are right angles, what are BD, AE, CE, BE, DE?



$N_1 := 2 \qquad N_2 := 3 \qquad N_3 := 4$

$AB := N_1 \quad CD := N_2 \quad AC := N_3$

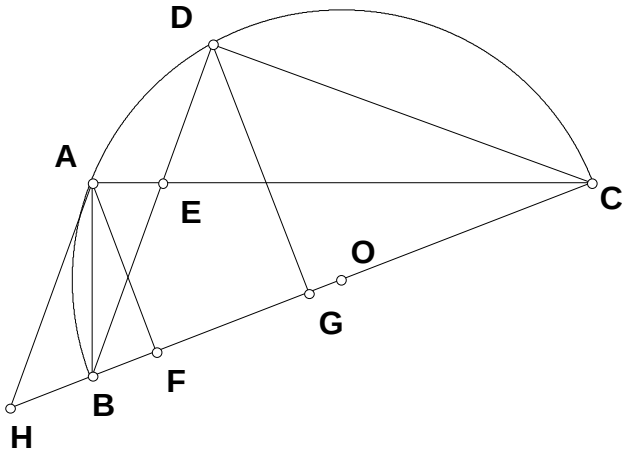
$BC := \sqrt{AB^2 + AC^2} \qquad CG := \frac{CD^2}{BC} \qquad BF := \frac{AB^2}{BC}$



$BG := BC - CG \qquad CF := BC - BF$

$AF := \sqrt{AB^2 - BF^2} \qquad DG := \sqrt{CD^2 - CG^2}$

$FH := \frac{BG \cdot AF}{DG} \qquad CH := CF + FH$



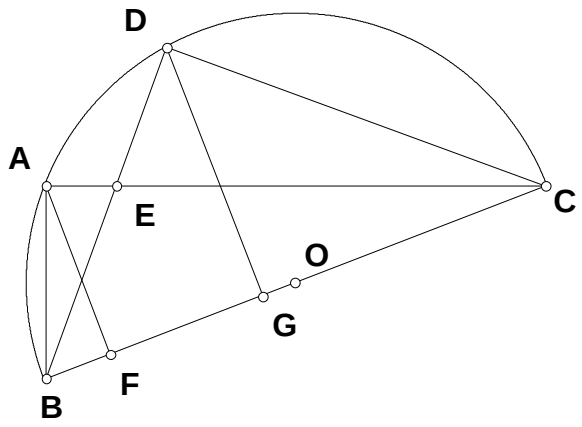
$BD := \sqrt{BG^2 + DG^2}$

$AH := \frac{BD \cdot FH}{BG} \qquad BE := \frac{AH \cdot BC}{CH}$

$DE := BD - BE \qquad CE := \frac{AC \cdot BC}{CH}$

$AE := AC - CE$

Idea about process progression: Given two differences one renders a set number of processes upon them, then one progresses to three?



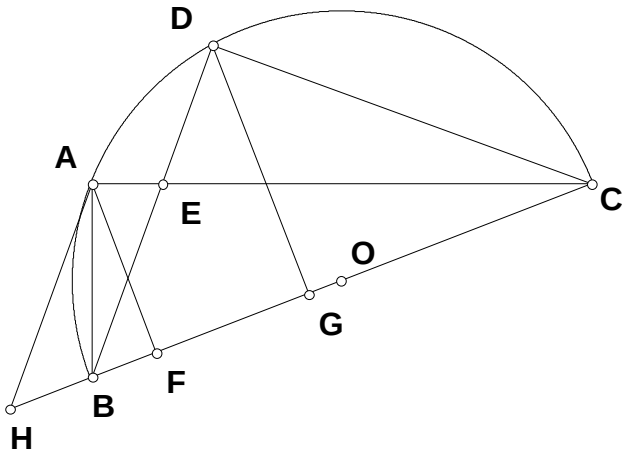
Some Algebraic Names:

$$\sqrt{N_1^2 + N_3^2} - BC = 0 \qquad \frac{N_2^2}{\sqrt{N_1^2 + N_3^2}} - CG = 0$$

$$\frac{N_1^2}{\sqrt{N_1^2 + N_3^2}} - BF = 0$$

$$\frac{\left(N_1^2 + N_3^2 - N_2^2\right)}{\sqrt{N_1^2 + N_3^2}} - BG = 0$$

$$\frac{N_3^2}{\sqrt{N_1^2 + N_3^2}} - CF = 0$$



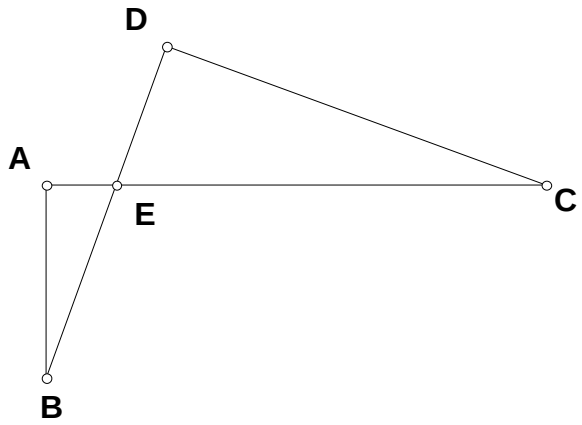
$$\frac{N_1 \cdot N_3}{\sqrt{N_1^2 + N_3^2}} - AF = 0$$

$$N_2 \cdot \sqrt{\frac{\left(N_1^2 + N_3^2 - N_2^2\right)}{\left(N_1^2 + N_3^2\right)}} - DG = 0$$

$$\frac{N_3 \cdot N_1 \cdot \left(N_1^2 + N_3^2 - N_2^2\right)}{N_2 \cdot \sqrt{\left(N_1^2 + N_3^2 - N_2^2\right) \cdot \left(N_1^2 + N_3^2\right)}} - FH = 0$$

$$\left[\frac{N_3^2}{\sqrt{N_1^2 + N_3^2}} + \frac{N_3 \cdot N_1 \cdot \left(N_1^2 + N_3^2 - N_2^2\right)}{N_2 \cdot \sqrt{\left(N_1^2 + N_3^2 - N_2^2\right) \cdot \left(N_1^2 + N_3^2\right)}}\right] - CH = 0 \qquad N_3 \cdot \frac{N_1}{N_2} - AH = 0$$

The Five Sought:



$$\sqrt{N_1^2 + N_3^2 - N_2^2} - BD = 0$$

$$N_2 \cdot \frac{\left(\sqrt{N_1^2 + N_3^2 - N_2^2} \cdot N_3 - N_1 \cdot N_2\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^2 + N_3^2 - N_2^2} \cdot N_1\right)} - DE = 0$$

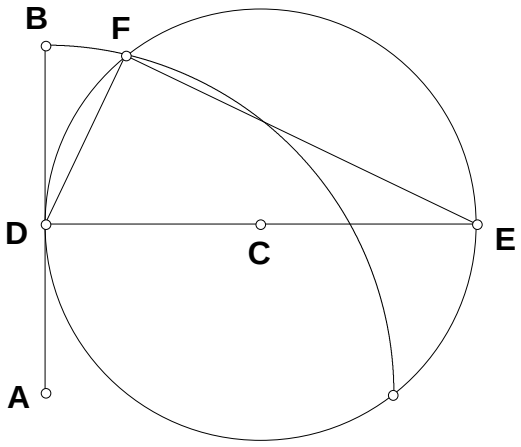
$$N_1 \cdot \frac{\left(N_1^2 + N_3^2\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^2 + N_3^2 - N_2^2} \cdot N_1\right)} - BE = 0$$

$$\frac{N_2 \cdot \left(N_1^2 + N_3^2\right)}{N_3 \cdot N_2 + \sqrt{N_1^2 + N_3^2 - N_2^2} \cdot N_1} - CE = 0$$

$$N_3 - \frac{N_2 \cdot \left(N_1^2 + N_3^2\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^2 + N_3^2 - N_2^2} \cdot N_1\right)} - AE = 0$$

042201

Given AB as unit, AD and DC, what is EF and DF?

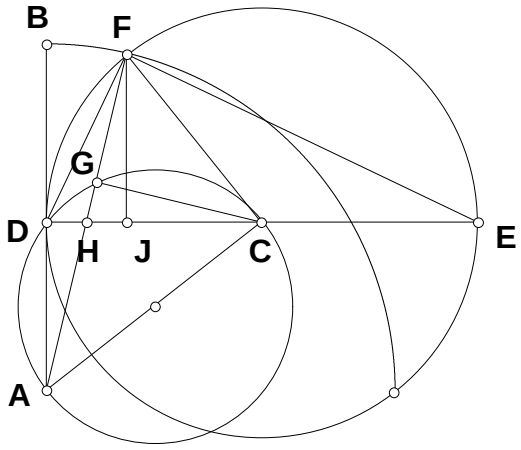


$N_1 := 2.052 \quad N_2 := .62 \quad AB := 1.802$

$AD := \frac{AB}{N_1} \quad CD := AB \cdot N_2 \quad DE := 2\ CD$

$AF := AB \quad CF := CD$

$AC := \sqrt{AD^2 + CD^2} \quad S_1 := AF \quad S_2 := AC \quad S_3 := CF$



$AG := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1} \quad CG := \sqrt{AC^2 - AG^2}$

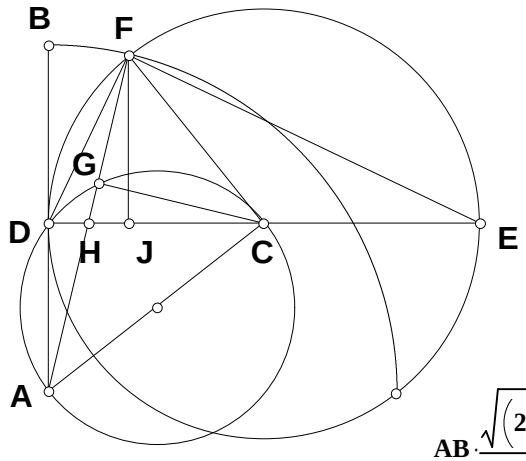
$L_1 := AD \quad L_2 := CG \quad L_3 := CD$

$$DH := L_3 - \frac{L_2 \cdot \left(L_1^2 + L_3^2\right)}{L_3 \cdot L_2 + \sqrt{L_1^2 + L_3^2 - L_2^2} \cdot L_1}$$

$$AH := L_1 \cdot \frac{\left(L_1^2 + L_3^2\right)}{\left(L_3 \cdot L_2 + \sqrt{L_1^2 + L_3^2 - L_2^2} \cdot L_1\right)} \quad FH := AF - AH \quad HJ := \frac{DH \cdot FH}{AH}$$

$DJ := DH + HJ \quad EJ := DE - DJ \quad FJ := \frac{AD \cdot FH}{AH} \quad EF := \sqrt{FJ^2 + EJ^2} \quad DF := \sqrt{DJ^2 + FJ^2}$

Some Algebraic Names:



$$\frac{AB}{N_1} - AD = 0 \qquad AB \cdot N_2 - CD = 0$$

$$(2\ AB) \cdot N_2 - DE = 0 \qquad AB \cdot \frac{\sqrt{\left(1 + N_2^{\,2} \cdot N_1^{\,2}\right)}}{N_1} - AC = 0$$

$$\frac{1}{2} \cdot AB \cdot \frac{\left(1 + N_1^{\,2}\right)}{N_1^{\,2}} - AG = 0$$

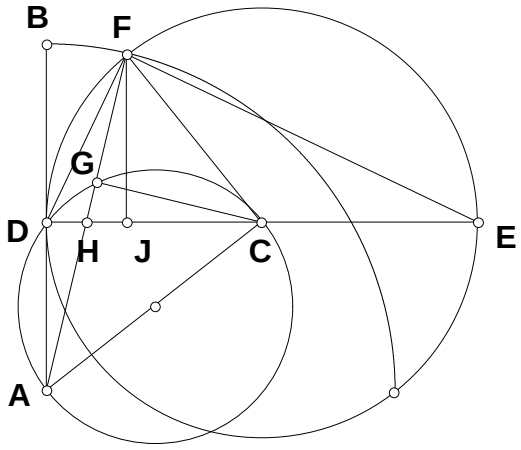
$$AB \cdot \frac{\sqrt{\left(2 \cdot N_1^{\,2} \cdot N_2 + N_1^{\,2} - 1\right) \cdot \left(2 \cdot N_1^{\,2} \cdot N_2 - N_1^{\,2} + 1\right)}}{2 \cdot N_1^{\,2}} - CG = 0$$

$$AB \cdot \frac{N_2 \cdot N_1^{\,3} + N_2 \cdot N_1 - \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}}}{N_1^{\,3} + N_1^{\,2} \cdot N_2 \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} + N_1} - DH = 0$$

$$2 \cdot AB \cdot \frac{\left(1 + N_2^{\,2} \cdot N_1^{\,2}\right)}{\left(N_1 \cdot N_2 \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} + 1 + N_1^{\,2}\right)} - AH = 0$$

$$AB - 2 \cdot AB \cdot \frac{\left(1 + N_2^{\,2} \cdot N_1^{\,2}\right)}{\left(N_1 \cdot N_2 \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} + 1 + N_1^{\,2}\right)} - FH = 0$$

$$\frac{-1}{2} \cdot AB \cdot \frac{\begin{bmatrix} -N_1^{\,4} \cdot N_2^{\,2} \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} \ \ldots \\ + 2 \cdot N_1^{\,3} \cdot N_2 - 2 \cdot N_1^{\,5} \cdot N_2 \ \ldots \\ + 6 \cdot N_1^{\,5} \cdot N_2^{\,3} - 3 \cdot N_2^{\,2} \cdot N_1^{\,2} \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} \ \ldots \\ + 2 \cdot N_2^{\,3} \cdot N_1^{\,3} - \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} \ \ldots \\ + N_1^{\,2} \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} \end{bmatrix}}{\left[N_1 \cdot \left[\left(N_1 \cdot N_2 \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} + 1 + N_1^{\,2}\right) \cdot \left(1 + N_2^{\,2} \cdot N_1^{\,2}\right)\right]\right]} - HJ = 0$$



$$\frac{1}{2}\cdot AB\cdot\frac{\left(N_1^{\,3}\cdot N_2+N_2\cdot N_1-\sqrt{2\cdot N_1^{\,2}+4\cdot N_1^{\,4}\cdot N_2^{\,2}-1-N_1^{\,4}}\right)}{\left[\left(1+N_2^{\,2}\cdot N_1^{\,2}\right)\cdot N_1\right]}-DJ=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{\left(3\cdot N_2\cdot N_1+4\cdot N_2^{\,3}\cdot N_1^{\,3}-N_1^{\,3}\cdot N_2+\sqrt{2\cdot N_1^{\,2}+4\cdot N_1^{\,4}\cdot N_2^{\,2}-1-N_1^{\,4}}\right)}{\left[\left(1+N_2^{\,2}\cdot N_1^{\,2}\right)\cdot N_1\right]}-EJ=0$$

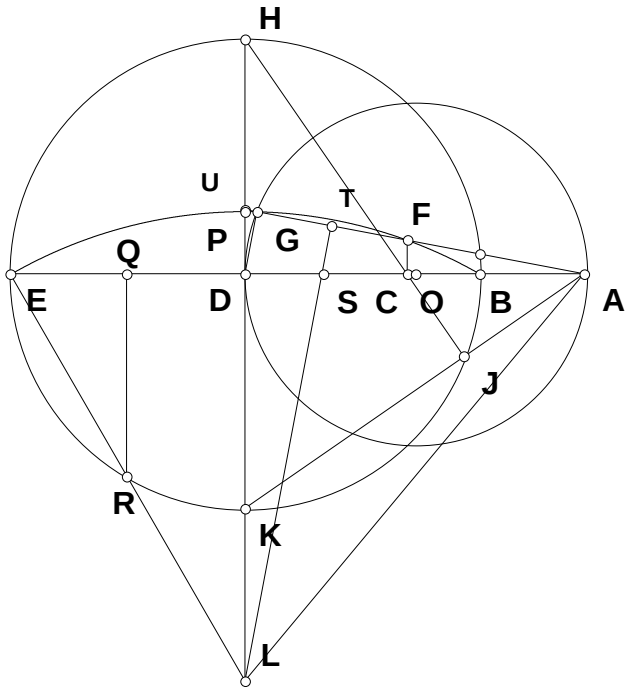
$$\frac{-1}{2}\cdot AB\,\frac{\left(-N_1\cdot N_2\cdot\sqrt{2\cdot N_1^{\,2}+4\cdot N_1^{\,4}\cdot N_2^{\,2}-1-N_1^{\,4}}+1-N_1^{\,2}+2\cdot N_2^{\,2}\cdot N_1^{\,2}\right)}{\left[N_1\cdot\left(1+N_2^{\,2}\cdot N_1^{\,2}\right)\right]}-FJ=0$$

$$AB\cdot\sqrt{N_2\cdot\frac{\left(3\cdot N_1\cdot N_2+4\cdot N_2^{\,3}\cdot N_1^{\,3}-N_1^{\,3}\cdot N_2+\sqrt{2\cdot N_1^{\,2}+4\cdot N_1^{\,4}\cdot N_2^{\,2}-1-N_1^{\,4}}\right)}{\left[\left(1+N_2^{\,2}\cdot N_1^{\,2}\right)\cdot N_1\right]}}-EF=0$$

$$AB\cdot\sqrt{\left(N_1^{\,3}\cdot N_2+N_1\cdot N_2-\sqrt{2\cdot N_1^{\,2}+4\cdot N_1^{\,4}\cdot N_2^{\,2}-1-N_1^{\,4}}\right)\cdot\frac{N_2}{\left[N_1\cdot\left(1+N_2^{\,2}\cdot N_1^{\,2}\right)\right]}}-DF=0$$

Counterpoint 042301

CF is the radius of a circle expressed in trisection. Is AF the counterpart of AC?



$N := 5.48643 \qquad AB := .54331$

$AE := AB \cdot N \qquad BE := AE - AB$

$BD := \frac{BE}{2} \qquad AD := AB + BD$

$AO := \frac{AD}{2} \qquad DE := BD \qquad DQ := \frac{DE}{2}$

$EQ := DQ \qquad BQ := BD + DQ$

$QR := \sqrt{BQ \cdot EQ} \qquad ER := \sqrt{QR^2 + EQ^2}$

$EL := 2 \cdot ER \qquad LP := EL \qquad DL := \frac{QR \cdot EL}{ER}$

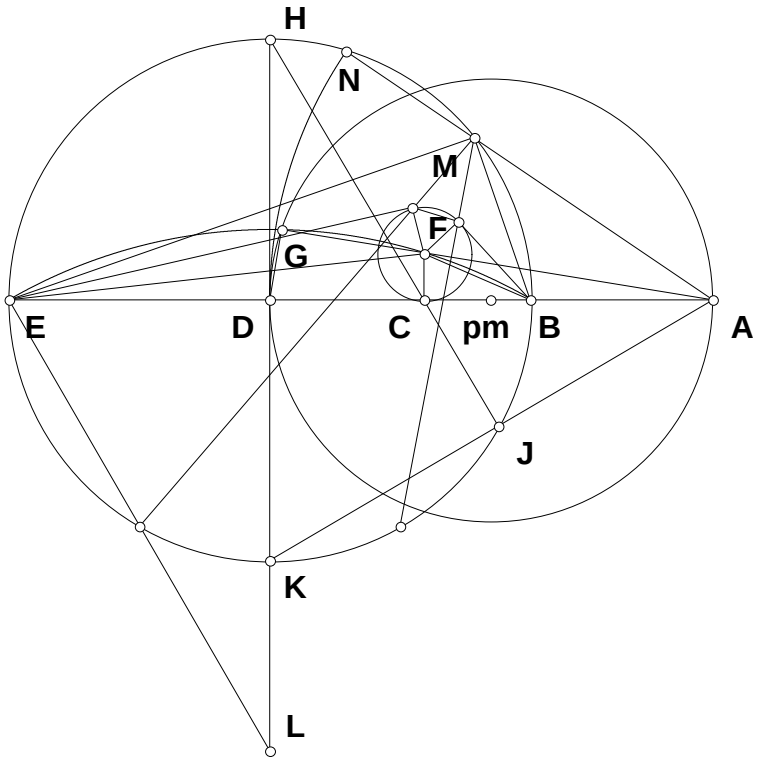
Try to figure out how I do this;

$$AG := \frac{AB \cdot (N-1)}{4} \cdot \sqrt{2} \cdot \sqrt{(1+N) \cdot \sqrt{3}} \cdot \sqrt{\frac{\left[\frac{5}{18} \cdot \sqrt{3} \cdot \frac{(1+N)}{(N-1)} + \frac{1}{18} \cdot \frac{(1+N)^3}{(N-1)^3} \cdot \sqrt{3} + \frac{1}{3} \cdot \frac{\sqrt{(N+3) \cdot (3 \cdot N+1)}}{(N-1)}\right]}{\left[\frac{1}{12} \cdot \frac{\left(13 \cdot N^2 - 22 \cdot N + 13\right)}{(N-1)}\right]}}$$

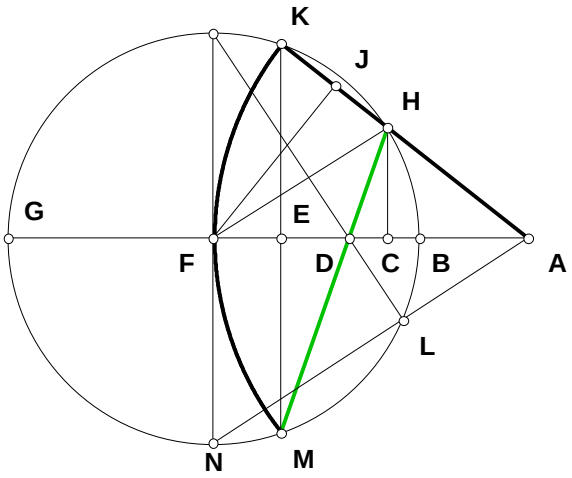
$DG := \sqrt{AD^2 - AG^2} \qquad AU := \frac{AD \cdot AD}{AG} \qquad DU := \frac{DG \cdot AD}{AG} \qquad LU := DL + DU \qquad TU := \frac{DG \cdot LU}{AD} \qquad GU := AU - AG$

$TG := TU - GU \qquad FT := TG \qquad AF := AG - (FT + TG) \qquad AC := \frac{AD \cdot AF}{AU}$

From 012496 for AC $2 \cdot AB \cdot \frac{N}{(1+N)}$ $AC - 2 \cdot AB \cdot \frac{N}{(1+N)} = 0$



042401



Does HM intersect at D? What is the Algebraic name of HM in relation to AB and AG?

$N := 5$ $AB := 1$ $AG := AB \cdot N$

$BG := AG - AB$ $BF := \frac{BG}{2}$ $AF := AB + BF$

$AK := AF$ $FK := BF$ $AE := \frac{2\,AK^2 - FK^2}{2\,AF}$

$AJ := AE$ $JK := AK - AJ$ $HJ := JK$

$AH := AK - (JK + HJ)$ $AC := \frac{AE \cdot AH}{AK}$ $CE := AE - AC$ $BE := AE - AB$ $EG := BG - BE$

$EK := \sqrt{BE \cdot EG}$ $BC := AC - AB$ $CG := BG - BC$ $CH := \sqrt{BC \cdot CG}$ $DE := \frac{CE \cdot EK}{EK + CH}$

$DF := 2 \cdot DE$ $HM := \sqrt{CE^2 + (EK + CH)^2}$

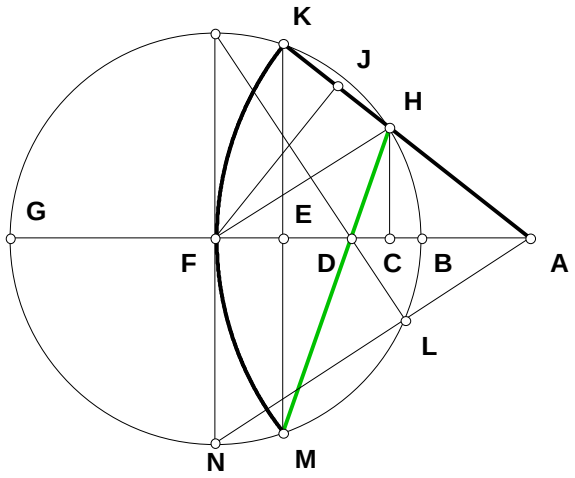
Some Algebraic Names:

$AB \cdot N - AB - BG = 0$ $\frac{1}{2} \cdot AB \cdot N - \frac{1}{2} \cdot AB - BF = 0$ $\frac{1}{2} \cdot AB + \frac{1}{2} \cdot AB \cdot N - AF = 0$

$\frac{1}{4} \cdot AB \cdot \frac{\left(N^2 + 6 \cdot N + 1\right)}{(1 + N)} - AE = 0$ $\frac{1}{4} \cdot AB \cdot \frac{\left(1 - 2 \cdot N + N^2\right)}{(1 + N)} - JK = 0$ $2 \cdot AB \cdot \frac{N}{(1 + N)} - AH = 0$

$AB \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)^3} \cdot N - AC = 0$ $\frac{1}{4} \cdot AB \cdot \left(N^2 + 6 \cdot N + 1\right) \cdot \frac{(N - 1)^2}{(1 + N)^3} - CE = 0$

$\frac{1}{4} \cdot AB \cdot (N + 3) \cdot \frac{(N - 1)}{(1 + N)} - BE = 0$ $\frac{1}{4} \cdot AB \cdot (3 \cdot N + 1) \cdot \frac{(N - 1)}{(1 + N)} - EG = 0$



$$\frac{1}{4}\cdot AB\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\frac{(N-1)}{(1+N)}-EK=0$$

$$AB\cdot(3\cdot N+1)\cdot\frac{(N-1)}{(1+N)^3}-BC=0$$

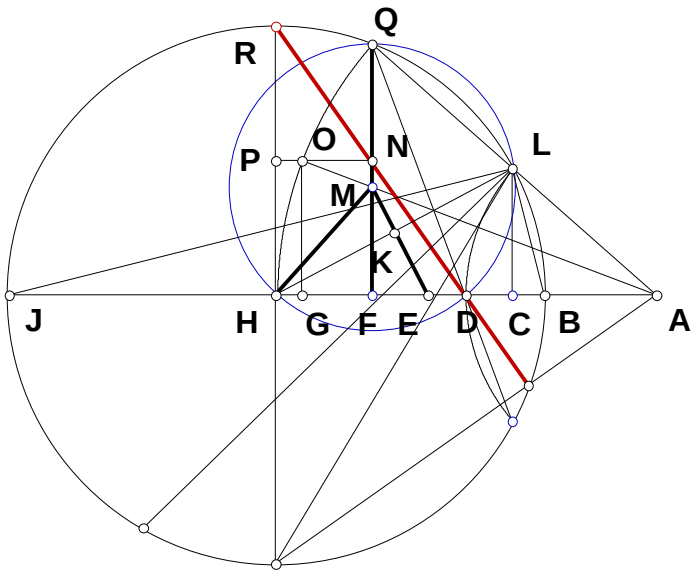
$$AB\cdot N^2\cdot(N+3)\cdot\frac{(N-1)}{(1+N)^3}-CG=0$$

$$AB\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\cdot\frac{(N-1)}{(1+N)^3}-CH=0$$

$$\frac{1}{4}\cdot AB\cdot\frac{(N-1)^2}{(1+N)}-DE=0 \qquad \frac{1}{2}\cdot AB\cdot\frac{(N-1)^2}{(N+1)}-DF=0 \qquad \frac{1}{2}\cdot AB\cdot(N-1)\cdot\frac{\left(1+6\cdot N+N^2\right)}{(1+N)^2}-HM=0$$

042501

What is the Algebraic name of the circle HM? Does point N divide DR in half?



$N := 5.768$ $AB := .583$ $AJ := AB \cdot N$

$BJ := AJ - AB$ $BH := \frac{BJ}{2}$ $HR := BH$

$HP := \frac{HR}{2}$ $GO := HP$ $AH := AB + BH$

$AO := AH$ $AG := \sqrt{AO^2 - GO^2}$

$HQ := BH$ $AQ := AH$ $FH := \frac{HQ^2}{2 \cdot AH}$

$AF := AH - FH$ $FM := \frac{GO \cdot AF}{AG}$

$HJ := BH$ $FJ := FH + HJ$ $BF := BJ - FJ$

$FQ := \sqrt{BF \cdot FJ}$ $MQ := FQ - FM$ $HM := \sqrt{FH^2 + FM^2}$ $HM - MQ = 0$

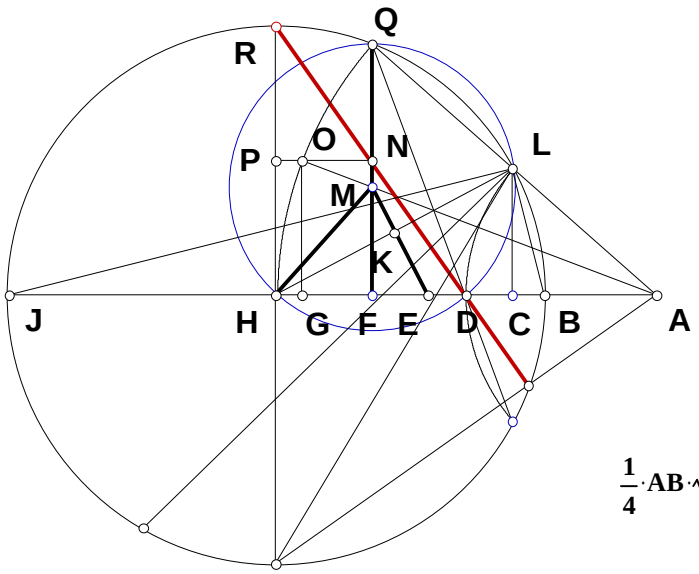
$DH := \frac{HR^2}{AH}$ $\frac{DH}{2} - FH = 0$

Some Algebraic Names:

$AB \cdot N - AB - BJ = 0$ $\frac{1}{2} \cdot AB \cdot (N - 1) - BH = 0$ $\frac{1}{4} \cdot AB \cdot (N - 1) - HP = 0$

$\frac{1}{2} \cdot AB \cdot (1 + N) - AH = 0$ $\frac{1}{4} \cdot AB \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} - AG = 0$ $\frac{1}{4} \cdot AB \cdot \frac{(N - 1)^2}{(1 + N)} - FH = 0$

$\frac{1}{4} \cdot AB \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)} - AF = 0$ $\frac{1}{4} \cdot (N - 1) \cdot AB \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{\left[(1 + N) \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)}\right]} - FM = 0$



$$\frac{1}{4}\cdot AB\cdot(N-1)\cdot\frac{(3\cdot N+1)}{(1+N)}-FJ=0$$

$$\frac{1}{4}\cdot AB\cdot(N+3)\cdot\frac{(N-1)}{(1+N)}-BF=0$$

$$\frac{1}{4}\cdot AB\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\frac{(N-1)}{(1+N)}-FQ=0$$

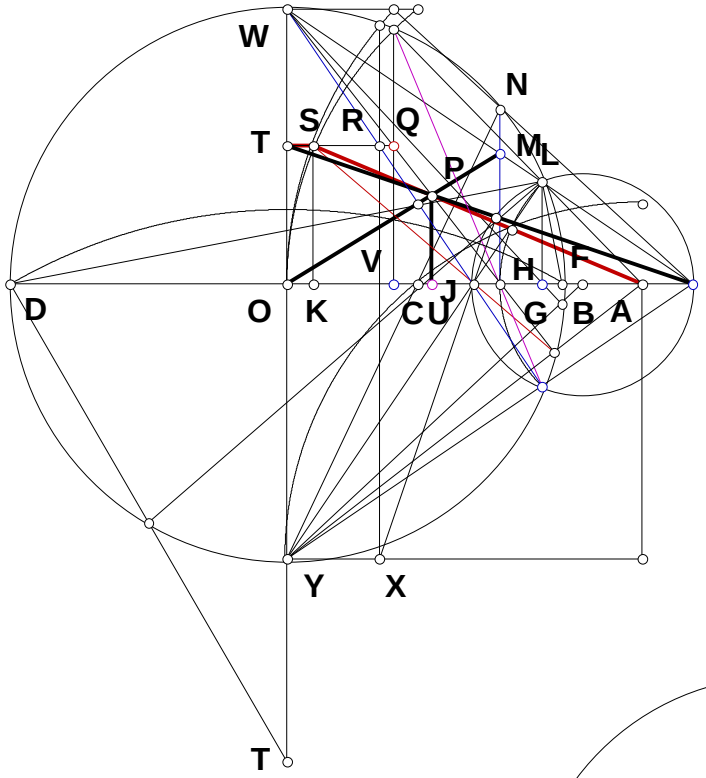
$$\frac{1}{2}\cdot(1+N)\cdot AB\cdot\frac{(N-1)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-MQ=0$$

$$\frac{1}{2}\cdot AB\cdot(N-1)\cdot\frac{(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-HM=0$$

$$HM-MQ=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{(N-1)^2}{(1+N)}-DH=0 \qquad \frac{DH}{2}-FH=0$$

Four Lines To A Point 042901

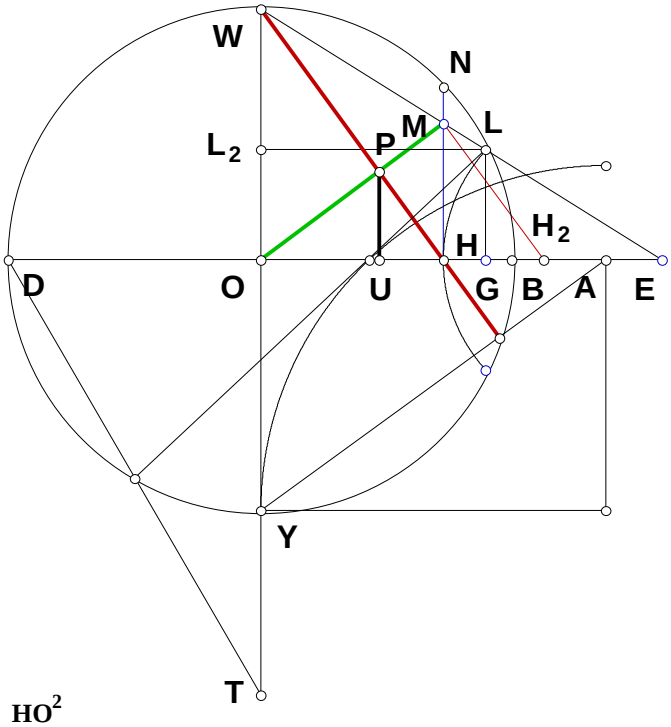


Does the difference OU and PU each have but one Algebraic name?

$N := 5 \qquad AB := 1 \qquad AD := AB \cdot N$

$BD := AD - AB \qquad BO := \frac{BD}{2} \qquad OW := BO$

$OY := BO \qquad DO := BO \qquad AO := AB + BO$



$HO := \frac{OY^2}{AO} \qquad AH := AO - HO \qquad AL := AH$

$OL := BO \qquad GO := \frac{OL^2 + AO^2 - AL^2}{2\,AO}$

$BG := BO - GO \qquad DG := GO + DO$

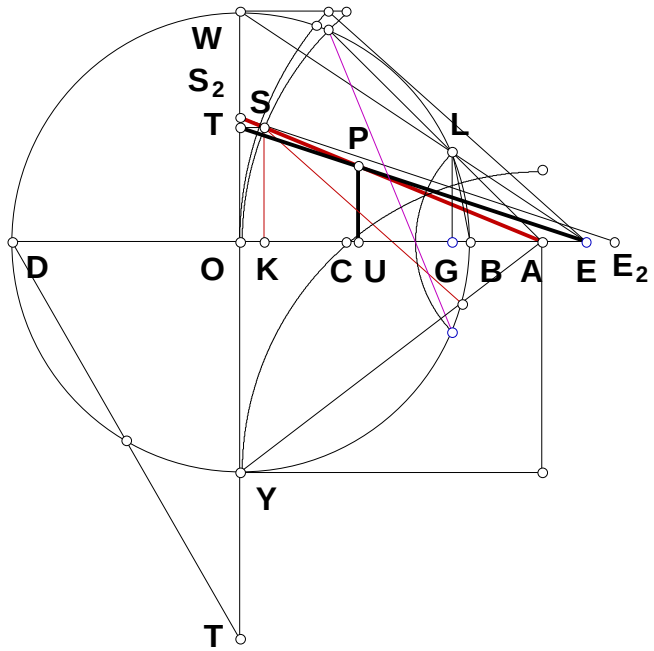
$GL := \sqrt{BG \cdot DG} \qquad EO := \frac{GO \cdot OW}{OW - GL}$

$BE := EO - BO \qquad BH := BO - HO$

$EG := BE + BG \qquad EH := BE + BH$

$HM := \frac{GL \cdot EH}{EG} \qquad HH_2 := \frac{HO \cdot HM}{OW} \qquad OU := \frac{HO^2}{HH_2 + HO}$

$UP := \frac{HM \cdot OU}{HO}$



$$OT := \frac{OW}{2} \qquad KS := OT \qquad AS := AO$$

$$AK := \sqrt{AS^2 - KS^2} \qquad OS_2 := \frac{KS \cdot AO}{AK}$$

$$OE_2 := \frac{EO \cdot OS_2}{OT} \qquad AS_2 := \sqrt{AO^2 + OS_2{}^2}$$

$$AE_2 := OE_2 - AO \qquad AE := EO - AO$$

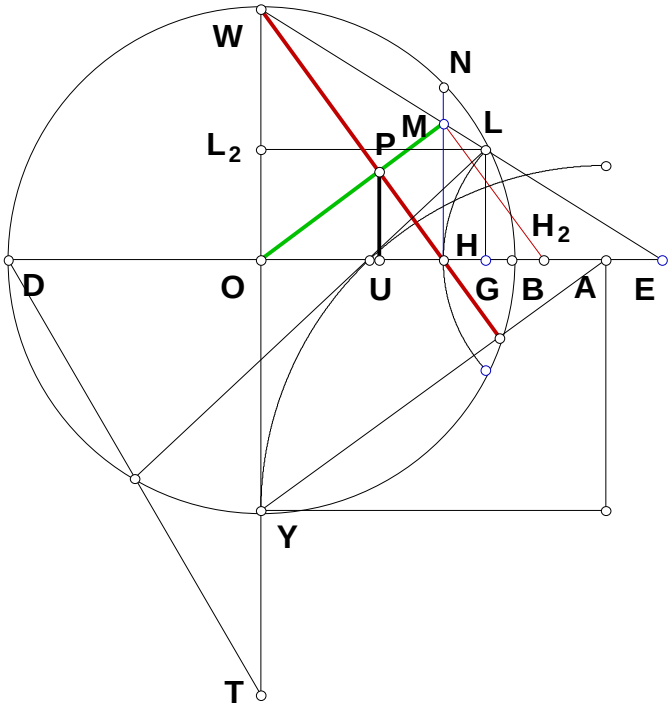
$$AP := \frac{AS_2 \cdot AE}{AE_2} \qquad AU := \frac{AO \cdot AE}{AE_2}$$

$$UO := AO - AU \qquad PU := \frac{OS_2 \cdot AU}{AO}$$

$$UO - OU = 0$$

$$PU - UP = 0$$

Some Algebraic Names:



$$AB\cdot(N-1)-BD=0$$

$$\frac{AB\cdot(N-1)}{2}-BO=0$$

$$\frac{1}{2}\cdot AB\cdot(1+N)-AO=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{(N-1)^2}{(1+N)}-HO=0$$

$$2\cdot AB\cdot\frac{N}{(1+N)}-AH=0$$

$$\frac{1}{2}\cdot AB\cdot\left(N^2+4\cdot N+1\right)\cdot\frac{(N-1)^2}{(1+N)^3}-GO=0$$

$$AB\cdot(3\cdot N+1)\cdot\frac{(N-1)}{(1+N)^3}-BG=0$$

$$AB\cdot(N-1)\cdot N^2\cdot\frac{(N+3)}{(1+N)^3}-DG=0$$

$$\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\cdot(N-1)\cdot\frac{AB}{(1+N)^3}-GL=0$$

$$\frac{1}{2}\cdot AB\cdot\left(N^2+4\cdot N+1\right)\cdot\frac{(N-1)^2}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}-EO=0$$

$$AB\cdot(N-1)\cdot\frac{\left[\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N-3\cdot N-1\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}-BE=0$$

$$AB\cdot\frac{(N-1)}{(1+N)}-BH=0$$

$$AB\cdot\frac{(N-1)^2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\cdot\left(N^2+4\cdot N+1\right)}{\left[\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]\cdot(1+N)^3\right]}-EG=0$$

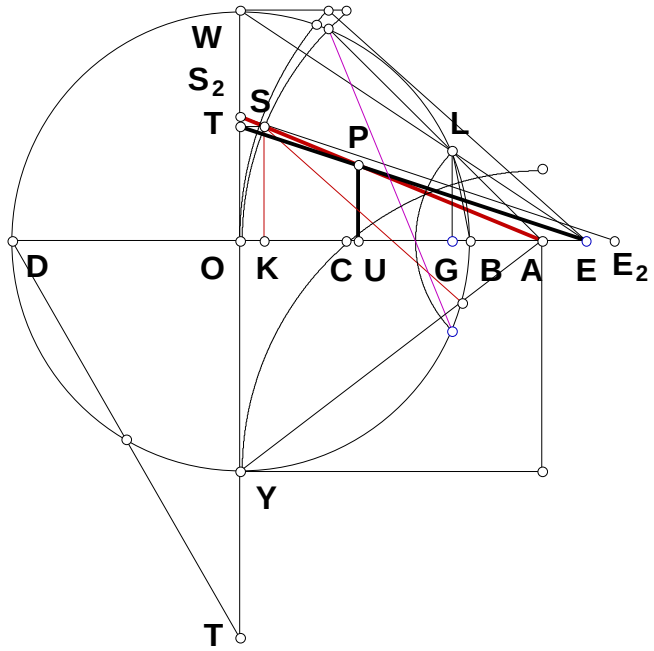
$$AB\cdot(N-1)^2\cdot N\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left[\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]\cdot(1+N)\right]}-EH=0$$

$$N\cdot(N-1)\cdot\frac{AB}{(1+N)}\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left(N^2+4\cdot N+1\right)}-HM=0$$

$$AB\cdot\frac{(N-1)^2}{(1+N)^2}\cdot N\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left(N^2+4\cdot N+1\right)}-HH_2=0$$

$$\frac{1}{2}\cdot AB\cdot(N-1)^2\cdot\frac{\left(N^2+4\cdot N+1\right)}{\left[7\cdot N\cdot(N+1)+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-OU=0$$

$$N\cdot(N-1)\cdot AB\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left[7\cdot N+7\cdot N^2+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-UP=0$$



$$\frac{1}{4}\cdot AB\cdot(N-1)-OT=0$$

$$\frac{AB}{4}\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}-AK=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{(N-1)\cdot(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-OS_2=0$$

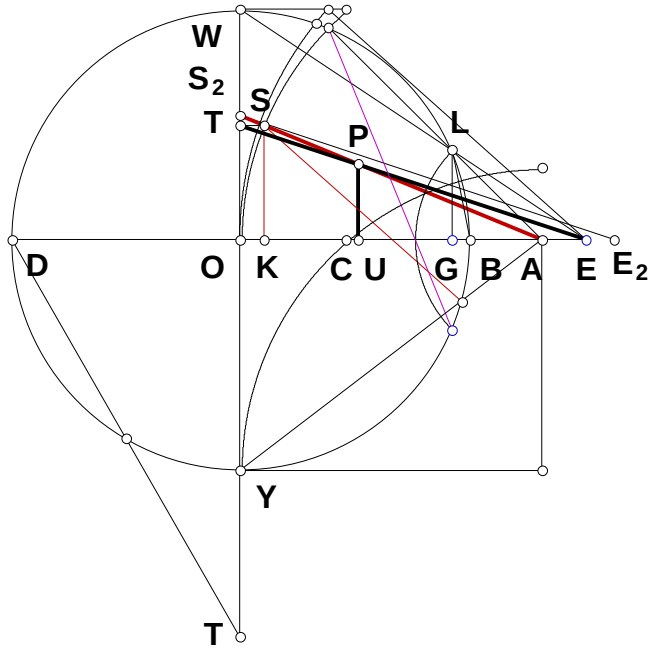
$$\frac{\left[AB\cdot\left(N^2+4\cdot N+1\right)\cdot(N-1)^2\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}\cdot\frac{(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-OE_2=0$$

$$(1+N)^2\cdot\frac{AB}{\sqrt{3+10\cdot N+3\cdot N^2}}-AS_2=0$$

$$\frac{\left[AB\cdot\left(N^2+4\cdot N+1\right)\cdot(N-1)^2\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}\cdot\frac{(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-\frac{1}{2}\cdot AB\cdot(1+N)-AE_2=0$$

$$-AB\cdot N\cdot\frac{\left[N^2+6\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}-AE=0$$

$$\frac{-2\cdot AB\cdot N\cdot(1+N)\cdot\left[N^2+6\cdot N-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}\right]}{\left(2\cdot N^4+10\cdot N^3+8\cdot N^2+10\cdot N+2\right)-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\ \ldots}-AP=0$$
$$+\ -\sqrt{(N+3)\cdot(3\cdot N+1)}-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^2-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^3$$



$$\frac{-AB\cdot N\cdot\left[N^2+6\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{\left(2\cdot N^4+10\cdot N^3+8\cdot N^2+10\cdot N+2\right)-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\ \ldots}-AU=0$$
$$+\ -\sqrt{(N+3)\cdot(3\cdot N+1)}-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^2-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^3$$

$$\frac{1}{2}\cdot AB\cdot(N-1)^2\cdot\frac{\left(N^2+4\cdot N+1\right)}{\left[7\cdot N\cdot(N+1)+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-UO=0$$

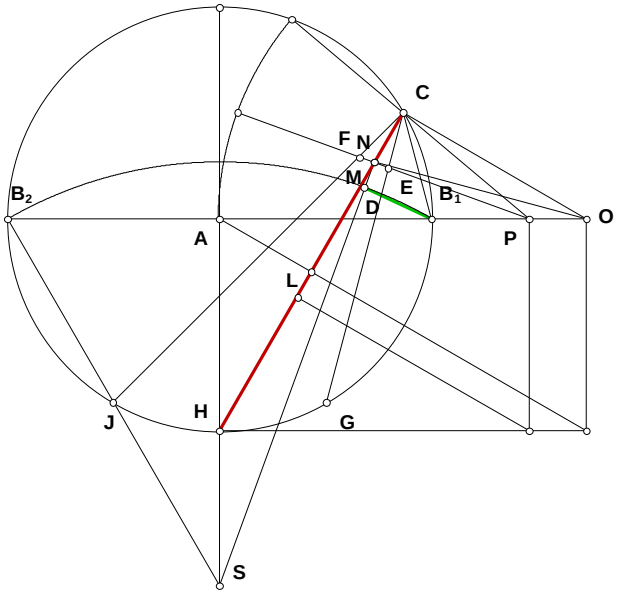
$$N\cdot(N-1)\cdot AB\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left[7\cdot N+7\cdot N^2+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-PU=0$$

Compass Construction 043001

$m\angle AB_1C = 75.000°$ $m\angle AB_2C = 15.000°$

$m\angle AB_1D = 25.000°$ $m\angle AB_2D = 5.000°$

$\frac{m\angle AB_1C}{m\angle AB_1D} = 3.000$ $\frac{m\angle AB_2C}{m\angle AB_2D} = 3.000$



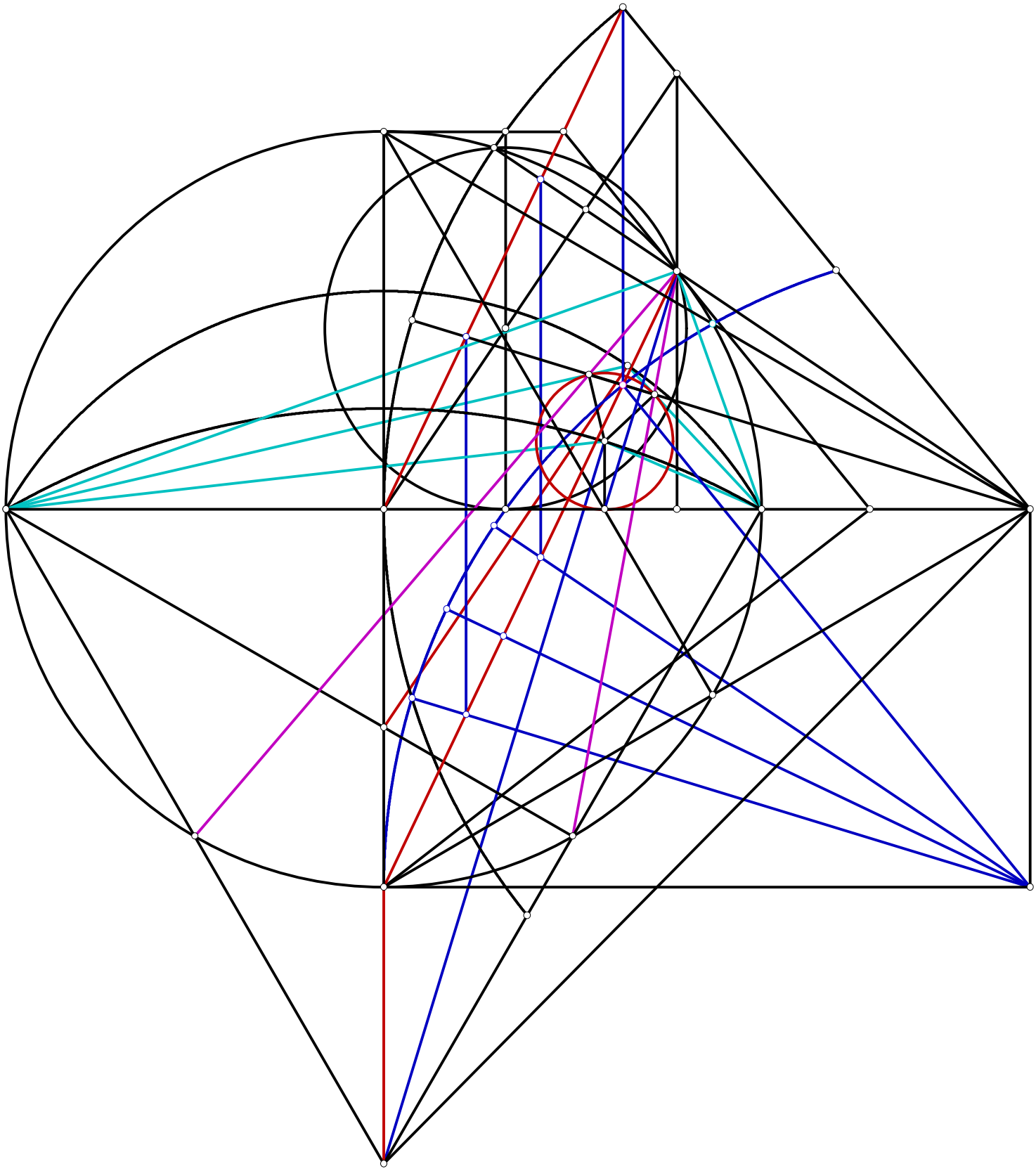
$m\angle ABC - 3 \cdot m\angle ABD = 0.000°$

$m\angle AB_2C - 3 \cdot m\angle AB_2D = -0.000°$

Move C->60 | Move C->75

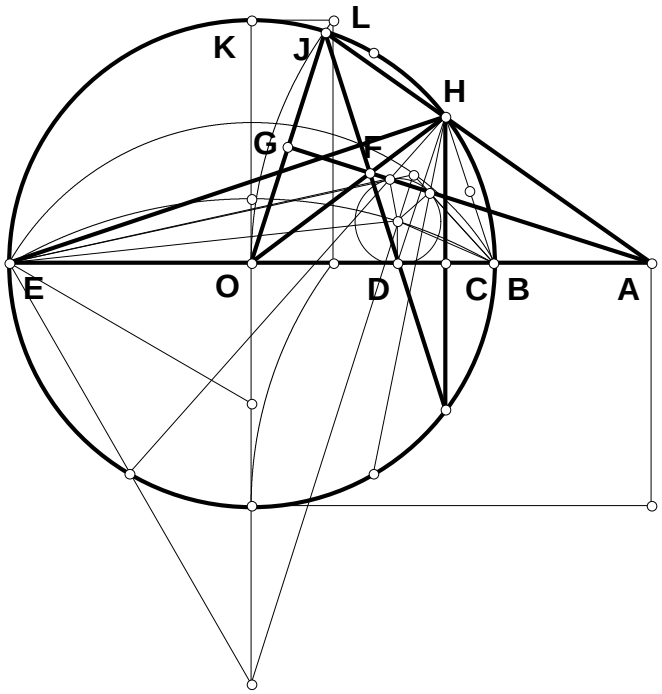
The process is simple, construct point O and then bisect AOC. From point N construct P from the perpendicular L-which is the midpoint of NS.
Bisect APC constructing E and F which are bisected at point M. Draw S to M to construct point D.

When there is no difference between two things, those things are said to be equal. This is true even of tolerance. If a figure is more accurate than the tools used to construct the figure, then that figure is equal, in that tool, to a solution that is tool independant.



050601.MCD

Just some Algebraic Names



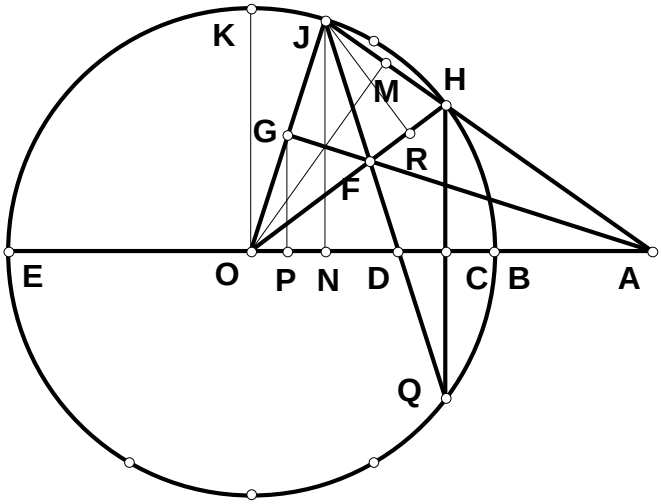
$AB := .818 \qquad N := 4.082$

$AE := AB \cdot N \qquad BE := AE - AB$

$BO := \frac{BE}{2} \qquad AO := AB + BO$

$AJ := AO \qquad JO := BO \qquad GO := \frac{JO}{2}$

$AG := \sqrt{AO^2 - GO^2}$



$AP := \frac{AG^2}{AO} \qquad OP := AO - AP$

$NO := 2 \cdot OP \qquad AN := AO - NO$

$JM := NO \qquad HO := BO$

$HJ := 2 \cdot JM \qquad AH := AJ - HJ$

$AC := \frac{AN \cdot AH}{AJ} \qquad CH := \sqrt{AH^2 - AC^2}$

$HQ := 2 \cdot CH \qquad CN := AN - AC$

$JN := \frac{CH \cdot AJ}{AH} \qquad CQ := CH$

$JQ := \sqrt{(CQ + JN)^2 + CN^2} \qquad OR := \frac{JO^2 + HO^2 - HJ^2}{2 \cdot HO} \qquad JR := \sqrt{JO^2 - OR^2}$

$FO := \frac{JO \cdot GO}{OR} \qquad FJ := FO \qquad DQ := \frac{JQ \cdot CQ}{CQ + JN} \qquad DF := JQ - (DQ + FJ) \qquad FH := HO - FO$

$FG := \frac{JR \cdot GO}{OR} \qquad AF := AG - FG$

$$AE - AB \cdot N = 0 \quad BE - (AB \cdot N - AB) = 0 \quad BO - \frac{1}{2} \cdot AB \cdot (N - 1) = 0$$

$$AO - \frac{1}{2} \cdot AB \cdot (1 + N) = 0 \qquad AG - \frac{AB}{4} \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} = 0$$

$$AP - \frac{AB}{8} \cdot (N + 3) \cdot \frac{(3 \cdot N + 1)}{(1 + N)} = 0 \qquad OP - \frac{AB}{8} \cdot \frac{(N - 1)^2}{(1 + N)} = 0 \qquad NO - \frac{AB}{4} \cdot \frac{(N - 1)^2}{(1 + N)} = 0$$

$$AN - \frac{AB}{4} \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)} = 0 \qquad HJ - \frac{AB}{2} \cdot \frac{(N - 1)^2}{(1 + N)} = 0 \qquad AH - 2 \cdot AB \cdot \frac{N}{(1 + N)} = 0$$

$$AC - AB \cdot \left(1 + 6 \cdot N + N^2\right) \cdot \frac{N}{(1 + N)^3} = 0 \qquad CH - AB \cdot N \cdot \sqrt{N + 3} \cdot \sqrt{3 \cdot N + 1} \cdot \frac{(N - 1)}{(1 + N)^3} = 0$$

$$HQ - 2 \cdot AB \cdot N \cdot \sqrt{N + 3} \cdot \sqrt{3 \cdot N + 1} \cdot \frac{(N - 1)}{(1 + N)^3} = 0 \quad CN - \frac{AB}{4} \cdot \left(N^2 + 6 \cdot N + 1\right) \cdot \frac{(N - 1)^2}{(1 + N)^3} = 0$$

$$JN - \frac{AB}{4} \cdot (N - 1) \cdot \sqrt{3 \cdot N + 1} \cdot \frac{\sqrt{N + 3}}{(1 + N)} = 0 \qquad JQ - \frac{AB}{2} \cdot (N - 1) \cdot \frac{\left(N^2 + 6 \cdot N + 1\right)}{(1 + N)^2} = 0$$

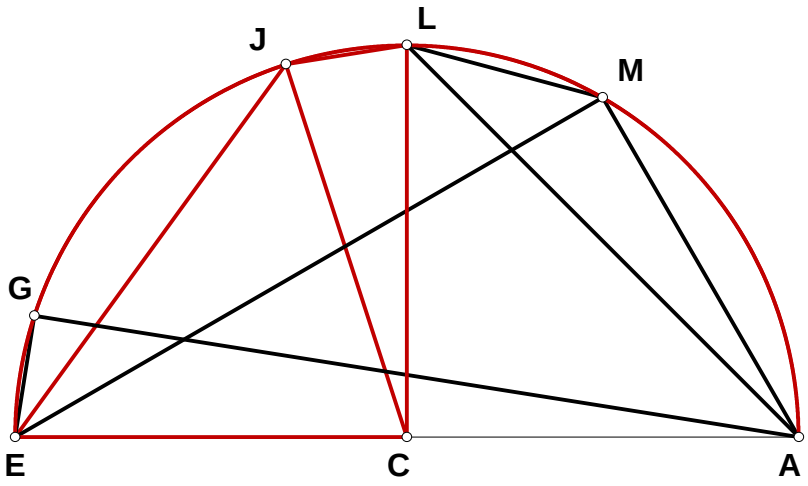
$$OR - \frac{AB}{4} \cdot \left(N^2 + 6 \cdot N + 1\right) \cdot \frac{(N - 1)}{(1 + N)^2} = 0 \qquad JR - \frac{AB}{4} \cdot (N - 1)^2 \cdot \sqrt{N + 3} \cdot \frac{\sqrt{3 \cdot N + 1}}{(1 + N)^2} = 0$$

$$FO - \frac{AB}{2} \cdot (1 + N)^2 \cdot \frac{(N - 1)}{\left(N^2 + 6 \cdot N + 1\right)} = 0 \qquad DQ - 2 \cdot N \cdot (N - 1) \cdot \frac{AB}{(1 + N)^2} = 0$$

$$DF - 2 \cdot N \cdot (N - 1) \cdot \frac{AB}{\left(N^2 + 6 \cdot N + 1\right)} = 0 \qquad FH - 2 \cdot N \cdot (N - 1) \cdot \frac{AB}{\left(N^2 + 6 \cdot N + 1\right)} = 0$$

$$FG - \frac{AB}{4} \cdot (N - 1)^2 \cdot \sqrt{N + 3} \cdot \frac{\sqrt{3 \cdot N + 1}}{\left(N^2 + 6 \cdot N + 1\right)} = 0 \qquad GO - \frac{AB}{4} \cdot (N - 1) = 0$$

$$AF - \left[\frac{AB}{4} \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} - \frac{1}{4} \cdot AB \cdot (N - 1)^2 \cdot \sqrt{N + 3} \cdot \frac{\sqrt{3 \cdot N + 1}}{\left(N^2 + 6 \cdot N + 1\right)} \right] = 0$$



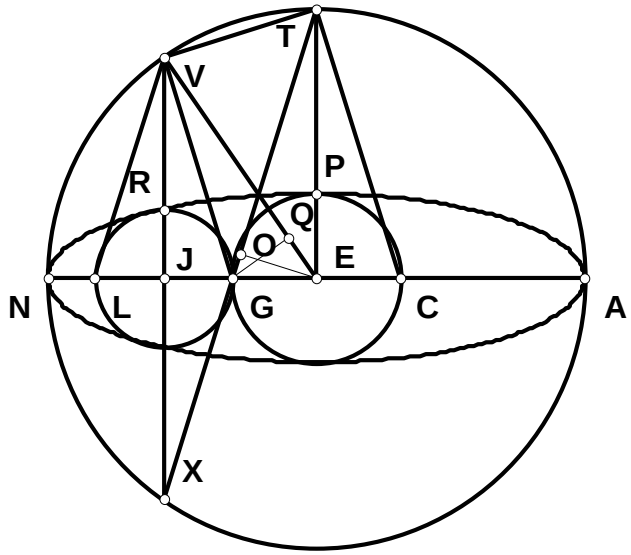
$$EM := \frac{1}{2}\cdot\sqrt{3} \qquad AL := \frac{1}{2}\cdot\sqrt{2} \qquad JL := \frac{1}{4}\cdot\sqrt{8 - 2\cdot\sqrt{10 + 2\cdot\sqrt{5}}} \qquad LM := \frac{1}{4}\cdot\sqrt{6} - \frac{1}{4}\cdot\sqrt{2}$$

$$AM := \frac{1}{2} \qquad EJ := \frac{1}{4}\cdot\sqrt{10 - 2\cdot\sqrt{5}}$$

$$AG := \frac{1}{4}\cdot\sqrt{8 + 2\cdot\sqrt{10 + 2\cdot\sqrt{5}}}$$

Elliptic Progression Outtake Two 0507012

Angles TEV and EVJ equals CTG.



$N := 6.381 \quad AN := 2.792$

$EN := \frac{AN}{2} \quad ET := EN \quad EV := EN$

$EG := \frac{AN}{N} \quad EP := EG \quad GT := \sqrt{EG^2 + ET^2}$

$GO := \frac{EG^2}{GT} \quad GX := GT - 2 \cdot GO$

$JX := \frac{ET \cdot GX}{GT} \quad GJ := \frac{EG \cdot GX}{GT}$

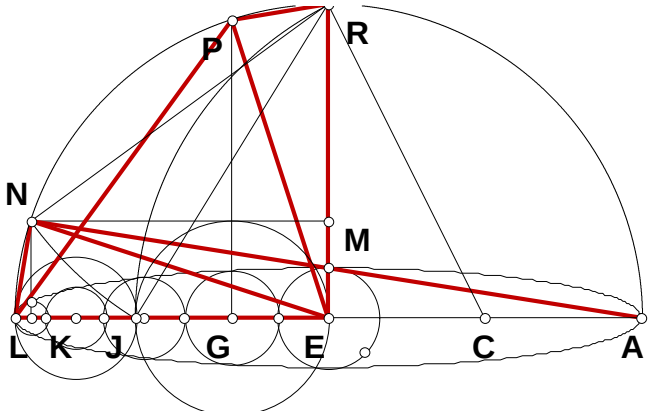
$\frac{ET}{EP} - \frac{JX}{GJ} = 0 \qquad JV := JX$

$EJ := EG + GJ$

$TV := \sqrt{ET^2 - 2 \cdot ET \cdot JV + JV^2 + EJ^2}$

$\frac{ET}{TV} - \frac{GT}{2 \cdot EG} = 0 \qquad EQ := \frac{EJ \cdot EG}{EV} \quad OQ := \frac{JV \cdot EQ}{EJ} \quad OQ - GJ = 0$

Outtake Three: Alternate Method: Pentasection Or Irrational Rationals 0507013



$AL := 1 \qquad AE := \frac{AL}{2} \qquad AC := \frac{AE}{2}$

$CE := AC \quad ER := AE \quad CR := \sqrt{CE^2 + ER^2}$

$CJ := CR \qquad EJ := CJ - CE$

$JR := \sqrt{EJ^2 + ER^2} \quad NR := JR$

$EN := AE \quad EM := \frac{EN^2 + ER^2 - NR^2}{2 \cdot ER}$

$KN := EM \quad EK := \sqrt{EN^2 - KN^2} \quad EL := AE \qquad KL := EL - EK \qquad LN := \sqrt{KL^2 + KN^2}$

$EG := \frac{EJ}{2} \qquad GL := EL - EG \quad AG := AE + EG \qquad GP := \sqrt{AG \cdot GL} \qquad PR := \sqrt{ER^2 - 2 \cdot ER \cdot GP + GP^2 + EG^2}$

$PR - LN = 0 \qquad AN := \sqrt{AL^2 - LN^2}$

Irrational **means the inability of a grammar to provide a name for a given thing. Many things are then irrational in Arithmetic due to the principles of the naming convention. Algebraic naming solves the problem by incorporating operands as part of a name. Algebra provides a degree of rationality then that is not achievable by Arithmetic. The following are some Algebraic names.**

$\frac{1}{2} - AE = 0 \qquad \frac{1}{4} - AC = 0 \qquad \frac{1}{4} \cdot \sqrt{5} - CR = 0 \qquad \frac{1}{4} \cdot \sqrt{5} - \frac{1}{4} - EJ = 0 \qquad \frac{1}{4} \cdot \sqrt{10 - 2 \cdot \sqrt{5}} - JR = 0$

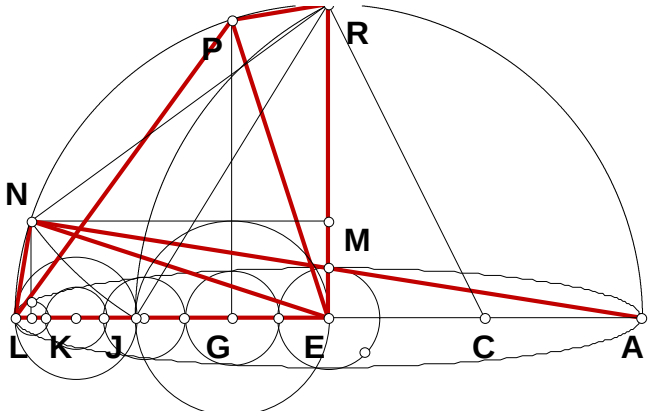
$\frac{-1}{8} + \frac{1}{8} \cdot \sqrt{5} - EM = 0 \qquad \frac{1}{2} \cdot \sqrt{\frac{5}{8} + \frac{1}{8} \cdot \sqrt{5}} - EK = 0 \qquad \frac{1}{2} - \frac{1}{8} \cdot \sqrt{10 + 2 \cdot \sqrt{5}} - KL = 0$

$\frac{1}{4} \cdot \sqrt{8 - 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - LN = 0 \qquad \frac{-1}{8} + \frac{1}{8} \cdot \sqrt{5} - EG = 0 \qquad \frac{5}{8} - \frac{1}{8} \cdot \sqrt{5} - GL = 0$

$\frac{3}{8} + \frac{1}{8} \cdot \sqrt{5} - AG = 0 \qquad \frac{1}{8} \cdot \sqrt{10 + 2 \cdot \sqrt{5}} - GP = 0 \qquad \frac{1}{4} \cdot \sqrt{8 - 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - PR = 0$

$\frac{1}{4} \cdot \sqrt{8 + 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - AN = 0$

Outtake Three: Alternate Method: Pentasection Or Irrational Rationals 0507013



$AL := 1 \qquad AE := \frac{AL}{2} \qquad AC := \frac{AE}{2}$

$CE := AC \quad ER := AE \quad CR := \sqrt{CE^2 + ER^2}$

$CJ := CR \qquad EJ := CJ - CE$

$JR := \sqrt{EJ^2 + ER^2} \quad NR := JR$

$EN := AE \quad EM := \frac{EN^2 + ER^2 - NR^2}{2 \cdot ER}$

$KN := EM \quad EK := \sqrt{EN^2 - KN^2} \quad EL := AE \qquad KL := EL - EK \qquad LN := \sqrt{KL^2 + KN^2}$

$EG := \frac{EJ}{2} \qquad GL := EL - EG \quad AG := AE + EG \qquad GP := \sqrt{AG \cdot GL} \qquad PR := \sqrt{ER^2 - 2 \cdot ER \cdot GP + GP^2 + EG^2}$

$PR - LN = 0 \qquad AN := \sqrt{AL^2 - LN^2}$

Irrational **means the inability of a grammar to provide a name for a given thing. Many things are then irrational in Arithmetic due to the principles of the naming convention. Algebraic naming solves the problem by incorporating operands as part of a name. Algebra provides a degree of rationality then that is not achievable by Arithmetic. The following are some Algebraic names.**

$\frac{1}{2} - AE = 0 \qquad \frac{1}{4} - AC = 0 \qquad \frac{1}{4} \cdot \sqrt{5} - CR = 0 \qquad \frac{1}{4} \cdot \sqrt{5} - \frac{1}{4} - EJ = 0 \qquad \frac{1}{4} \cdot \sqrt{10 - 2 \cdot \sqrt{5}} - JR = 0$

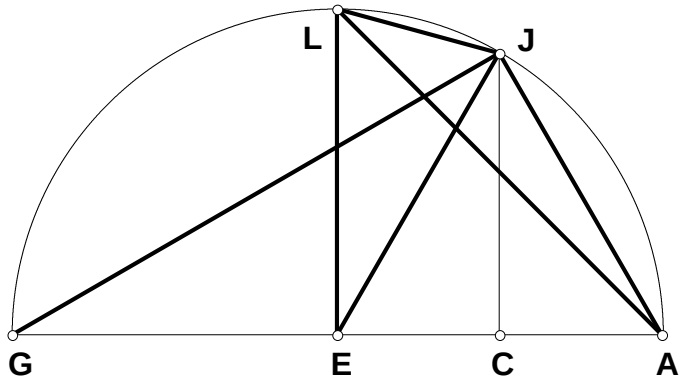
$\frac{-1}{8} + \frac{1}{8} \cdot \sqrt{5} - EM = 0 \qquad \frac{1}{2} \cdot \sqrt{\frac{5}{8} + \frac{1}{8} \cdot \sqrt{5}} - EK = 0 \qquad \frac{1}{2} - \frac{1}{8} \cdot \sqrt{10 + 2 \cdot \sqrt{5}} - KL = 0$

$\frac{1}{4} \cdot \sqrt{8 - 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - LN = 0 \qquad \frac{-1}{8} + \frac{1}{8} \cdot \sqrt{5} - EG = 0 \qquad \frac{5}{8} - \frac{1}{8} \cdot \sqrt{5} - GL = 0$

$\frac{3}{8} + \frac{1}{8} \cdot \sqrt{5} - AG = 0 \qquad \frac{1}{8} \cdot \sqrt{10 + 2 \cdot \sqrt{5}} - GP = 0 \qquad \frac{1}{4} \cdot \sqrt{8 - 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - PR = 0$

$\frac{1}{4} \cdot \sqrt{8 + 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - AN = 0$

Outtake Four: Some Names 0507014



$AG := 1 \quad AE := \frac{AG}{2} \quad AC := \frac{AE}{2}$

$CG := AG - AC \quad CJ := \sqrt{AC \cdot CG}$

$EL := AE \quad CE := AC$

$JL := \sqrt{EL^2 - 2 \cdot EL \cdot CJ + CJ^2 + CE^2}$

$AJ := \sqrt{AC^2 + CJ^2} \quad GJ := \sqrt{CG^2 + CJ^2}$

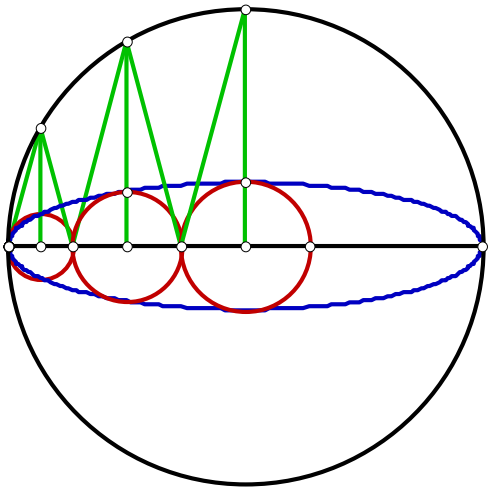
$AL := \sqrt{AE^2 + EL^2}$

$AE := \frac{1}{2} \quad AC := \frac{1}{4} \quad CG := 1 - \frac{1}{4} \quad CJ := \frac{1}{4} \cdot \sqrt{3} \quad JL := \frac{1}{4} \cdot \sqrt{6} - \frac{1}{4} \cdot \sqrt{2}$

$AJ := \frac{1}{2} \quad GJ := \frac{1}{2} \cdot \sqrt{3}_{*} \quad AL := \frac{1}{2} \cdot \sqrt{2}$

Elliptic Progression

050701

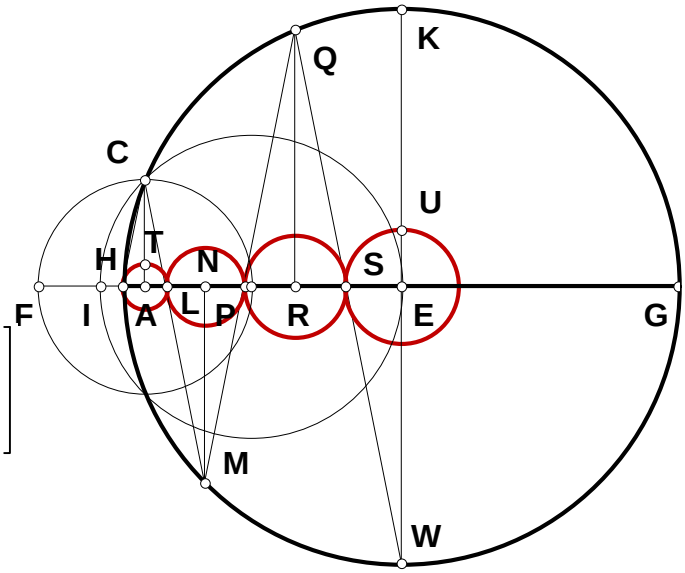


An Elliptic Progression takes place on a finite length of line. An Elliptic Progression may be defined in terms of a number of diameters of smaller circles, each defined by the same angle from the circumferance of the larger circle, from the center of a circle to its perimeter. When the sum of the number of those diameters minus one half the starting diameter are equal to the radius of the larger circle, the angle that defined the smaller circles will divide the larger circle evenly and the same number of times as the total number of smaller circles. This means that the division of a circle into equal angles is based on a simple elliptic funciton.

$\Delta := 2 \qquad N := 0 .. \Delta$

$AC_0 := .552 \qquad CP_0 := \sqrt{2 \left(AC_0\right)^2}$

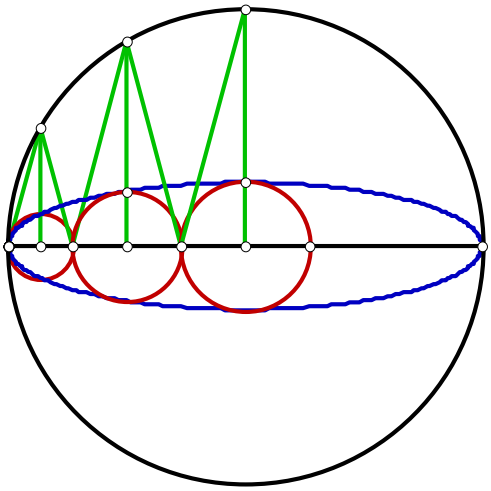
$$\begin{bmatrix} AC_{N+1} \\ CP_{N+1} \end{bmatrix} := \begin{bmatrix} CP_N \\ \sqrt{\left(AC_0\right)^2 + \left[\left(AC_N + CP\right)_N\right]^2} \end{bmatrix}$$

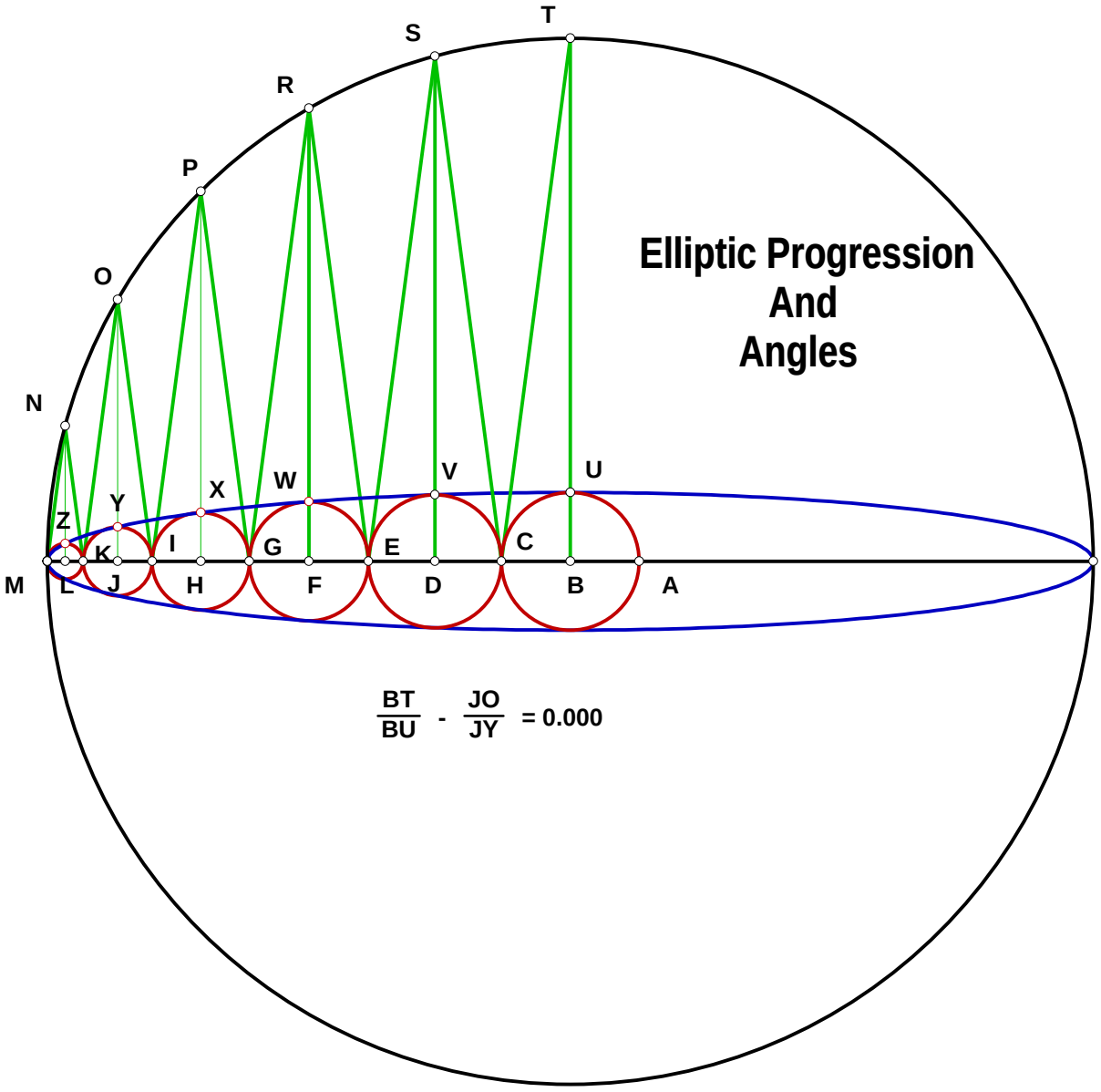


$CE := AC_\Delta \qquad AE := \sqrt{CE^2 - \left(AC_0\right)^2}$

$EH := CE \quad EK := CE \quad AH := EH - AE \quad AT := AH \qquad EU := \frac{AH \cdot EK}{AC_0}$

$EU = 0.287$




T
S
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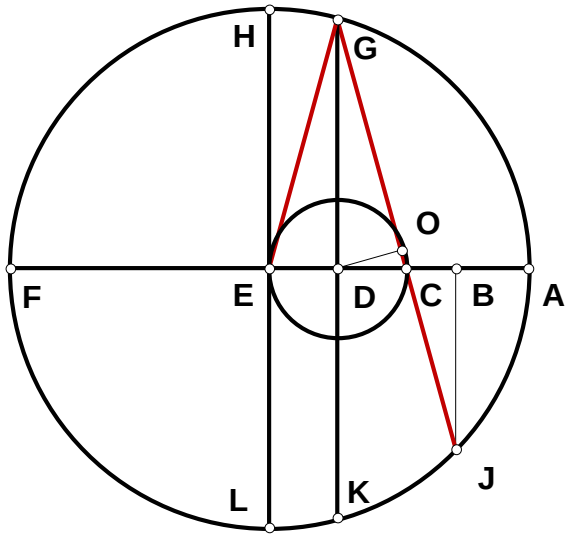
E
D
C
B
A
Z
Y
X
W
V
U
Elliptic Progression
And
Angles
BT/BU - JO/JY = 0.000

Elliptic Progression Outtake One 0507011



A method of trisection Algebraically.

$N := M$

$N \geq 4 = 1$ $\quad AF := 6$ $\quad AE := \frac{AF}{2}$

$DE := \frac{AF}{N}$ $\quad AD := AE - DE$ $\quad DF := AF - AD$

$DG := \sqrt{AD \cdot DF}$ $\quad CD := DE$ $\quad EG := AE$

$CO := \frac{CD^2}{EG}$ $\quad CG := EG$ $\quad CJ := CG - 4 \cdot CO$

$BC := \frac{CD \cdot CJ}{CG}$ $\quad AB := AE - (2 \cdot DE + BC)$

$BJ := \frac{DG \cdot BC}{CD}$ $\quad BD := BC + CD$

$JK := \sqrt{DG^2 - 2 \cdot DG \cdot BJ + BJ^2 + BD^2}$ $\quad \frac{JK}{2 \cdot DE} = 1$ **Some Algebraic Names,**

Part of this demonstration may be something of a reductio ad absurdum, if one supposed that CJ were not true. I suppose I need a plate to demonstrate it.

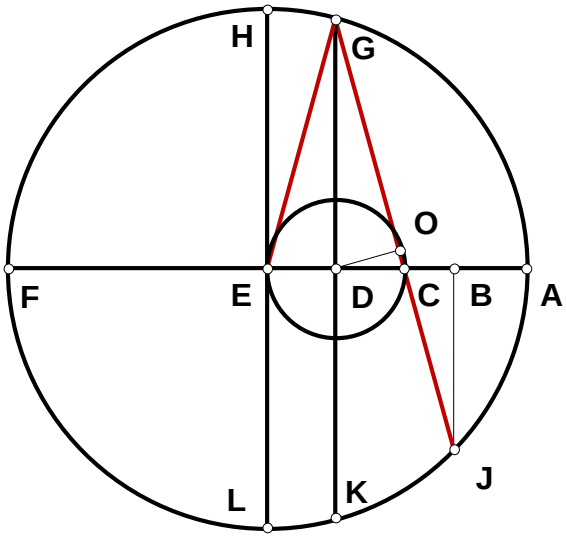
$AF \cdot \frac{(N-2)}{2 \cdot N} - AD = 0$ $\quad AF \cdot \frac{(N+2)}{2 \cdot N} - DF = 0$ $\quad AF \cdot \frac{\sqrt{(N-2) \cdot (N+2)}}{2 \cdot N} - DG = 0$

$\frac{2\ AF}{N^2} - CO = 0$ $\quad AF \cdot \frac{(N-4) \cdot (N+4)}{2 \cdot N^2} - CJ = 0$ $\quad AF \cdot \frac{(N-4) \cdot (N+4)}{N^3} - BC = 0$

$AF \cdot \frac{(N+2) \cdot (N-4)^2}{2 \cdot N^3} - AB = 0$ **One of the meanings of trisection is solving for the following equation when given AF and AB.**

$$\frac{AF}{AB} - \frac{2 \cdot N^3}{(N+2) \cdot (N-4)^2} = 0$$

$AF \cdot \frac{(N-4) \cdot (N+4) \cdot \sqrt{(N-2) \cdot (N+2)}}{2 \cdot N^3} - BJ = 0$ $\quad AF \cdot \frac{2 \cdot (N^2 - 8)}{N^3} - BD = 0$ $\quad \frac{2 \cdot AF}{N} - JK = 0$



$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2} - \frac{AF}{AB} = 0 \qquad \frac{AF}{AB} = 6$$

$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2} - 6 = 0$$

$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2} = 6$$

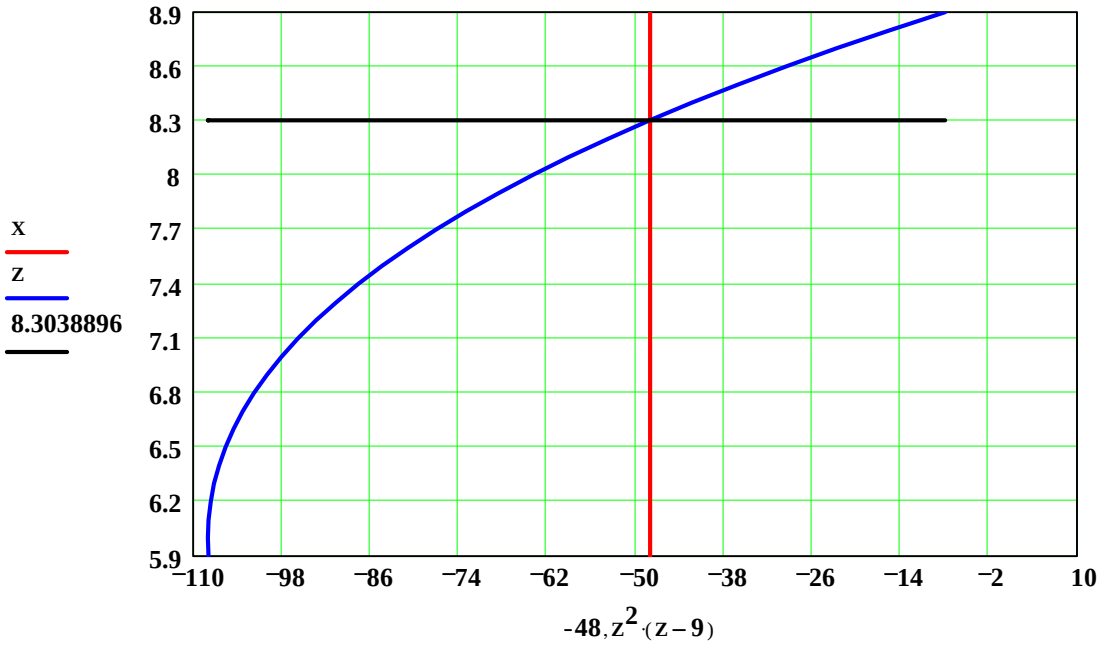
$$2\cdot M^3 - 6\cdot\left[(M+2)\cdot(M-4)^2\right] = 0$$

$$2\cdot M^3 - \left(6\cdot M^3 - 36\cdot M^2 + 192\right) = 0$$

$$4\cdot M^3 - 36\cdot M^2 + 192 = 0$$

$$M^3 - 9\cdot M^2 + 48 = 0 \qquad M^2\cdot(M-9) + 48 = 0 \qquad M \equiv 8.303889634816388$$

$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2} = 6 \qquad DE - \frac{AF}{M} = 0 \qquad Z := 5.9, 6 .. 8.9$$



Angles and the Ellipse 050801.mcd

$AC := 1 \qquad CP := \sqrt{2\ AC^2}$

$AP := AC \qquad EP := CP$

$CE := \sqrt{AC^2 + (AP + EP)^2}$

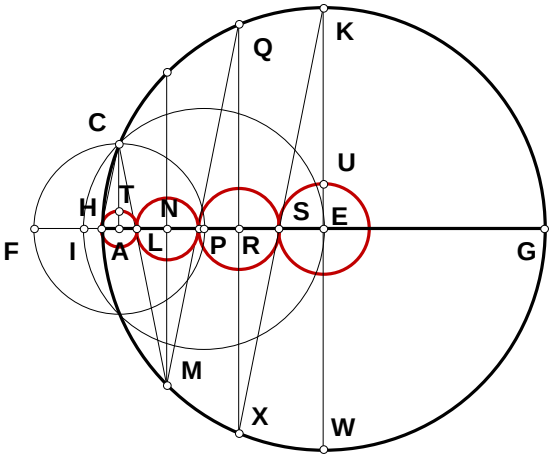
$EK := CE \quad EH := CE$

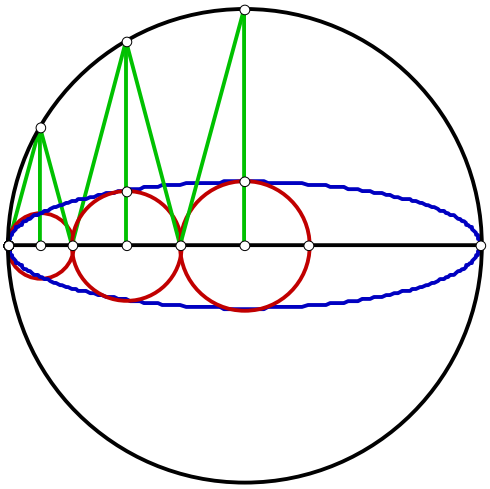
$AE := \sqrt{CE^2 - AC^2}$

$AH := EH - AE$

$AT := AH \qquad EU := \frac{AT \cdot EK}{AC}$

$\frac{EK}{EU} - \left(1 + \sqrt{2} + \sqrt{2} \cdot \sqrt{2 + \sqrt{2}}\right) = 0$

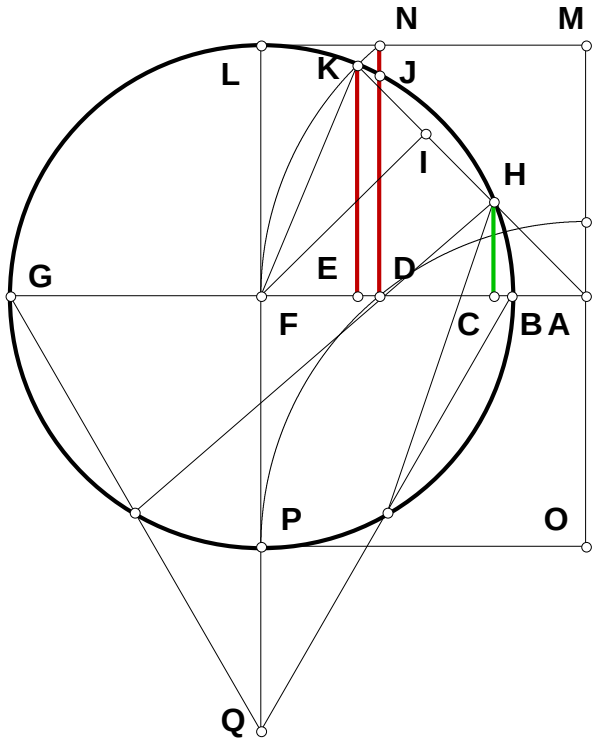




An Elliptic Progression takes place on a finite length of line. An Elliptic Progression may be defined in terms of a number of diameters of smaller circles, each defined by the same angle from the circumferance of the larger circle and from the center of a circle to its perimeter. When the sum of the number of those diameters minus one half the starting diameter are equal to the radius of the larger circle, the angle that defined the smaller circles will divide the larger circle evenly and the same number of times as the total number of smaller circles. This means that the division of a circle into equal angles may be expressed as an elliptic function.

On Trisection 051301

For any given trisection what is the Algebraic names of BC and BE taking BG as unit?



$N := 9 \qquad BG := 1 \qquad AG := BG \cdot N$

$AB := AG - BG \qquad BF := \frac{BG}{2}$

$AF := AB + BF \qquad AN := AF \qquad AK := AN$

$FK := BF \qquad S_1 := AF \qquad S_2 := AK \qquad S_3 := FK$

$AE := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1} \qquad AI := AE$

$IK := AK - AI \qquad HI := IK \qquad AH := AK - (HI + IK)$

$AC := \frac{AE \cdot AH}{AK} \qquad BC := AC - AB \qquad BE := AE - AB$

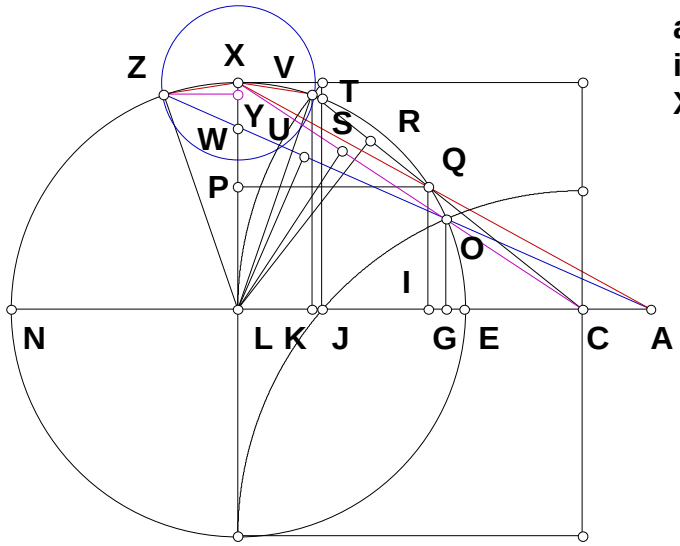
Some Algebraic Names:

$N - 1 - AB = 0 \qquad \frac{1}{2} - BF = 0 \qquad \frac{1}{2} \cdot (2 \cdot N - 1) - AF = 0 \qquad \frac{1}{4} \cdot \frac{\left(8 \cdot N^2 - 8 \cdot N + 1\right)}{(2 \cdot N - 1)} - AE = 0$

$\frac{1}{4} \cdot \frac{1}{(2 \cdot N - 1)} - IK = 0 \qquad 2 \cdot N \cdot \frac{(N - 1)}{(2 \cdot N - 1)} - AH = 0 \qquad \frac{\left(8 \cdot N^2 - 8 \cdot N + 1\right)}{(2 \cdot N - 1)^3} \cdot N \cdot (N - 1) - AC = 0$

$(N - 1)^2 \cdot \frac{(4 \cdot N - 1)}{(2 \cdot N - 1)^3} - BC = 0 \qquad \frac{1}{4} \cdot \frac{(4 \cdot N - 3)}{(2 \cdot N - 1)} - BE = 0$

On Trisection 051401.MCD



For any given QLX, XLZ is 1/3 of that angle. What are the Algebraic names in this figure for the cords QX and XZ?

$N := 5 \quad CE := 1 \quad CN := CE \cdot N$

$EN := CN - CE \quad EL := \frac{EN}{2} \quad LV := EL$

$CL := CE + EL$

$CV := CL \quad S_1 := CL \quad S_2 := CV$

$S_3 := LV \quad CK := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1}$

$CR := CK \quad RV := CV - CR \quad QR := RV \quad CQ := CV - (QR + RV) \quad CI := \frac{CK \cdot CQ}{CV} \quad EI := CI - CE$

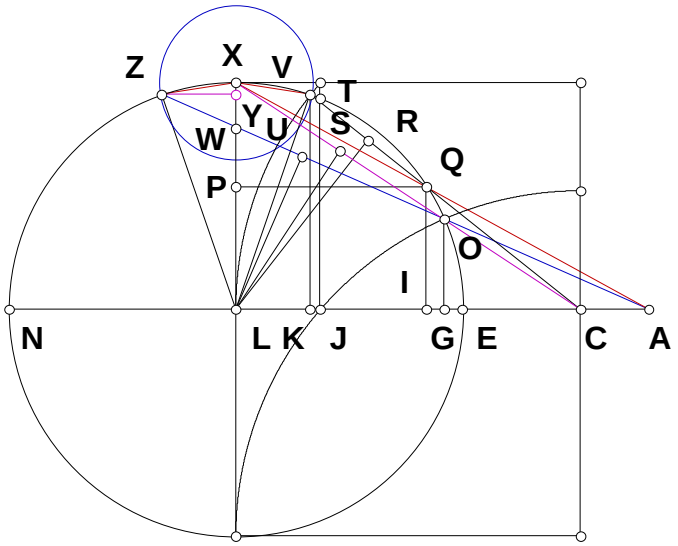
$IN := EN - EI \quad IQ := \sqrt{EI \cdot IN} \quad LP := IQ \quad LX := EL \quad IL := EL - EI \quad PX := LX - LP \quad AL := \frac{IL \cdot LX}{PX}$

$AE := AL - EL \quad AC := AE - CE \quad AI := AC + CI \quad AX := \sqrt{AL^2 + LX^2} \quad AQ := \frac{AX \cdot AI}{AL} \quad QX := AX - AQ$

$CX := \sqrt{CL^2 + LX^2} \quad CS := \frac{CL^2}{CX} \quad SX := CX - CS \quad OS := SX \quad CO := CX - (SX + OS)$

$GO := \frac{LX \cdot CO}{CX} \quad CG := \frac{CL \cdot CO}{CX} \quad EG := CG - CE \quad AG := AE + EG \quad AO := \sqrt{AG^2 + GO^2}$

$AU := \frac{AG \cdot AL}{AO} \quad OU := AU - AO \quad UZ := OU \quad AZ := AO + (OU + UZ)$



$$LW := \frac{GO \cdot AL}{AG} \qquad AW := \frac{AO \cdot AL}{AG}$$

$$WZ := AZ - AW \qquad WY := \frac{GO \cdot WZ}{AO}$$

$$YZ := \frac{AG \cdot WZ}{AO} \qquad YX := LX - (LW + WY)$$

$$XZ := \sqrt{YX^2 + YZ^2}$$

Some Algebraic Names:

$$\frac{1}{2} \cdot CE \cdot N - \frac{1}{2} \cdot CE - EL = 0 \qquad \frac{1}{4} \cdot CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)} - CK = 0 \qquad \frac{1}{4} \cdot CE \cdot \frac{\left(1 + N^2 - 2 \cdot N\right)}{(1 + N)} - RV = 0$$

$$2 \cdot CE \cdot \frac{N}{(1 + N)} - CQ = 0 \qquad CE \cdot \frac{N^3 + 6\,N^2 + N}{(N + 1)^3} - CE - EI = 0 \qquad CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)^3} \cdot N - CI = 0$$

$$CE \cdot N^2 \cdot \frac{\left(-3 + 2 \cdot N + N^2\right)}{(1 + N)^3} - IN = 0 \qquad (N - 1) \cdot N \cdot CE \cdot \frac{\sqrt{(N + 3) \cdot (3 \cdot N + 1)}}{(1 + N)^3} - IQ = 0$$

$$\frac{1}{2} \cdot CE \cdot \frac{\left(2 \cdot N - 6 \cdot N^2 + 2 \cdot N^3 + N^4 + 1\right)}{(1 + N)^3} - IL = 0$$

$$\left[\frac{1}{2} \cdot CE \cdot (N - 1) \cdot \frac{\left[N^3 + 3 \cdot N^2 - 2 \cdot N \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} + 3 \cdot N + 1\right]}{(1 + N)^3}\right] - PX = 0$$

$$\frac{1}{2}\cdot CE\cdot\left(N^2+4\cdot N+1\right)\cdot\frac{(N-1)^2}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}-AL=0$$

$$CE\cdot(N-1)\cdot\frac{\left[N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}-3\cdot N-1\right]}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}-AE=0$$

$$-CE\cdot N\cdot\frac{\left[-N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+6\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}+N^2\right]}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}-AC=0$$

$$\frac{CE\cdot(N-1)^2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\left(N^2+4\cdot N+1\right)}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]\cdot(1+N)^3}-AI=0$$

$$\frac{1}{2}\cdot CE\cdot(N-1)\cdot\sqrt{2}\cdot\sqrt{\frac{(1+N)^3}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}}-AX=0$$

$$CE\cdot(N-1)\cdot\sqrt{2}\cdot\sqrt{\frac{(1+N)^3}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}}\cdot N\cdot\frac{\sqrt{(N+3)\cdot(3\cdot N+1)}}{(1+N)^3}-AQ=0$$

$$\sqrt{\frac{\left[2\cdot(1+N)^3\right]}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\ \ldots\right.}}\cdot CE\cdot(N-1)\cdot\frac{\left[\left(N^3+3\cdot N^2\right)-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\ \ldots\right.}{2\cdot(1+N)^3}\ \ldots=0$$

$$\left.+3\cdot N+1\right]$$

$$\left.+3\cdot N+1\right]$$

$$+-QX$$

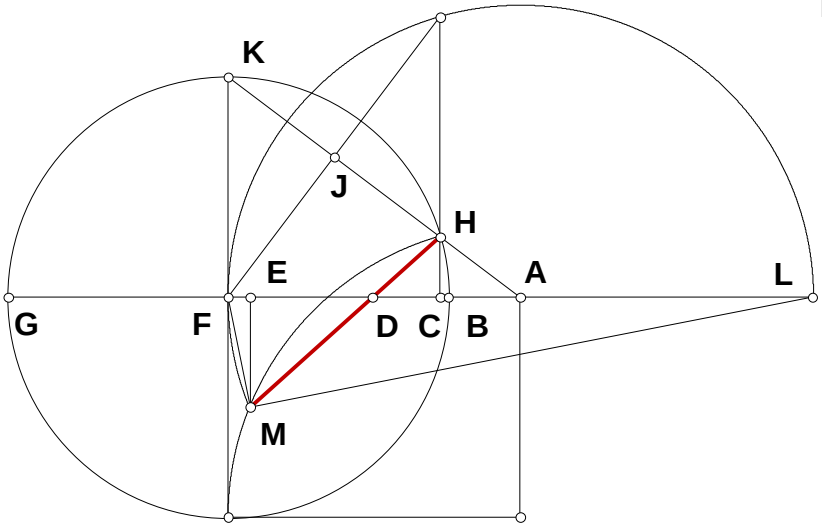
$$\frac{CE}{2}\cdot\sqrt{2}\cdot\sqrt{\left(1+N^2\right)}-CX=0 \qquad \frac{1}{4}\cdot CE\cdot(1+N)^2\cdot\frac{\sqrt{2}}{\sqrt{1+N^2}}-CS=0 \qquad \frac{1}{4}\cdot\sqrt{2}\cdot CE\cdot\frac{(N-1)^2}{\sqrt{1+N^2}}-SX=0$$

$$\frac{\sqrt{2}}{\sqrt{1+N^2}}\cdot CE\cdot N-CO=0 \qquad CE\cdot N\cdot\frac{(N-1)}{\left(1+N^2\right)}-GO=0 \qquad CE\cdot\frac{N^2+N}{\left(1+N^2\right)}-CG=0 \qquad CE\cdot\frac{(N-1)}{\left(1+N^2\right)}-EG=0$$

$$CE\cdot(N-1)^2\cdot N\cdot\frac{\left[N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}-2\cdot N+\sqrt{(N+3)\cdot(3\cdot N+1)}\right]}{\left[\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]\cdot\left(1+N^2\right)\right]}-AG=0$$

$$\frac{CE\cdot(N-1)}{2}\cdot\sqrt{\frac{\left[\begin{array}{l}\left[2\cdot N^4+14\cdot N^3-N^3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+32\cdot N^2\right]-7\cdot N^2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\ \ldots\\ +\,-7\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+14\cdot N+2-\sqrt{(N+3)\cdot(3\cdot N+1)}\end{array}\right]}{\left[(1+N)\cdot\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]\right]}}-XZ=0$$

Segment DF And HM 052201



Given AB and AG, what is HM and DF?

$N := 7.111$ $AB := .375$

$AG := AB \cdot N$ $BG := AG - AB$

$BF := \frac{BG}{2}$ $FK := BF$

$AF := AB + BF$

$AK := \sqrt{AF^2 + FK^2}$

$AJ := \frac{AF^2}{AK}$ $JK := AK - AJ$

$HJ := JK$ $AH := AK - (JK + HJ)$ $AC := \frac{AF \cdot AH}{AK}$ $EM := \frac{BF}{2}$ $FL := 2 \cdot AF$ $EF := \frac{FL}{2} - \frac{\sqrt{-4\ EM^2 + FL^2}}{2}$

$AE := AF - EF$ $CE := AE - AC$ $CH := \frac{FK \cdot AH}{AK}$ $HM := \sqrt{(EM + CH)^2 + CE^2}$ $DE := \frac{CE \cdot EM}{EM + CH}$

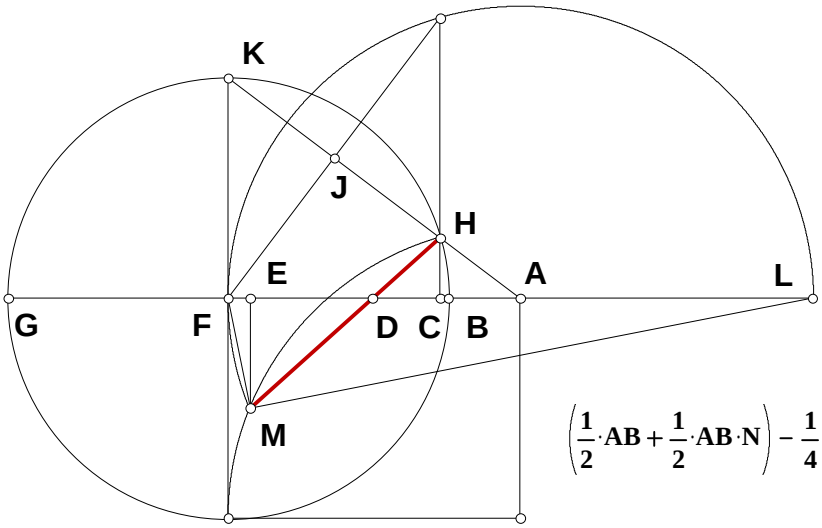
$DF := DE + EF$

Some Algebraic Names:

$AB \cdot N - AG = 0$ $AB \cdot N - AB - BG = 0$ $\frac{1}{2} \cdot AB \cdot N - \frac{1}{2} \cdot AB - BF = 0$ $\frac{1}{2} \cdot AB \cdot N - \frac{1}{2} \cdot AB - FK = 0$

$\frac{1}{2} \cdot AB + \frac{1}{2} \cdot AB \cdot N - AF = 0$ $\frac{1}{2} \cdot AB \cdot \sqrt{2} \cdot \sqrt{1 + N^2} - AK = 0$ $\frac{1}{4} \cdot AB \cdot (1 + N)^2 \cdot \frac{\sqrt{2}}{\sqrt{1 + N^2}} - AJ = 0$

$\frac{1}{4} \cdot \sqrt{2} \cdot AB \cdot \frac{(1 + N^2 - 2 \cdot N)}{\sqrt{1 + N^2}} - JK = 0$ $AB \cdot \sqrt{2} \cdot \frac{N}{\sqrt{1 + N^2}} - AH = 0$ $AB \cdot (1 + N) \cdot \frac{N}{1 + N^2} - AC = 0$



$$\frac{1}{4}\cdot AB\cdot N-\frac{1}{4}\cdot AB-EM=0$$

$$AB+AB\cdot N-FL=0$$

$$\left(\frac{1}{2}\cdot AB+\frac{1}{2}\cdot AB\cdot N\right)-\frac{1}{4}\cdot AB\cdot\sqrt{3\cdot N^2+10\cdot N+3}-EF=0$$

$$\frac{1}{4}\cdot AB\cdot\sqrt{3\cdot N^2+10\cdot N+3}-AE=0$$

$$\frac{1}{4}\cdot AB\cdot\frac{\left(\sqrt{3\cdot N^2+10\cdot N+3}+\sqrt{3\cdot N^2+10\cdot N+3}\cdot N^2-4\cdot N-4\cdot N^2\right)}{\left(1+N^2\right)}-CE=0$$

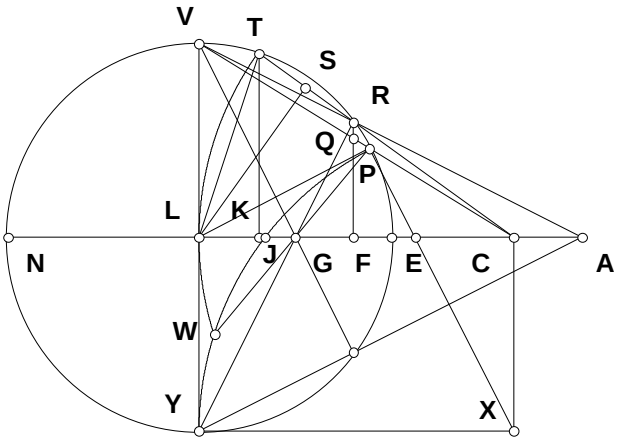
$$(N-1)\cdot AB\cdot\frac{N}{\left(1+N^2\right)}-CH=0$$

$$\frac{1}{2}\cdot AB\cdot\sqrt{(1+N)\cdot\frac{\left[N^3+3\cdot N^2-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+3\cdot N+1\right]}{\left(1+N^2\right)}}-HM=0$$

$$\frac{1}{4}\cdot\left(\sqrt{3\cdot N^2+10\cdot N+3}+\sqrt{3\cdot N^2+10\cdot N+3}\cdot N^2-4\cdot N-4\cdot N^2\right)\cdot\frac{AB}{\left(N^2+4\cdot N+1\right)}-DE=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{\left(3\cdot N+3\cdot N^2+1+N^3-2\cdot\sqrt{3\cdot N^2+10\cdot N+3}\cdot N\right)}{\left(N^2+4\cdot N+1\right)}-DF=0$$

Point of Intersection 052701



Do RY and PW intersect at G?

$N := 5 \qquad CE := 1 \qquad CN := CE \cdot N$

$EN := CN - CE \qquad EL := \frac{EN}{2} \qquad LV := EL$

$LT := EL \qquad LY := EL \qquad CL := CE + EL \qquad CT := CL$

$S_1 := CL \qquad S_2 := CT \qquad S_3 := LT$

$KL := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1} \qquad CK := CL - KL \qquad CS := CK \qquad ST := CT - CS \qquad RS := ST$

$CR := CT - (ST + RS) \qquad CF := \frac{CK \cdot CR}{CT} \qquad FR := \sqrt{CR^2 - CF^2} \qquad FQ := \frac{LV \cdot CF}{CL} \qquad FL := CL - CF$

$GL := \frac{FL \cdot LY}{(LY + FR)}$

Some Algebraic Names:

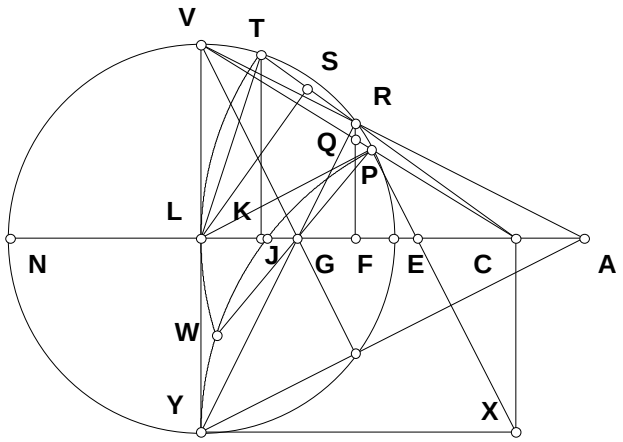
$CE \cdot N - CN = 0 \qquad CE \cdot N - CE - EN = 0 \qquad \frac{1}{2} \cdot CE \cdot N - \frac{1}{2} \cdot CE - EL = 0 \qquad \frac{1}{2} \cdot CE + \frac{1}{2} \cdot CE \cdot N - CL = 0$

$\frac{1}{4} \cdot CE \cdot \frac{(N-1)^2}{(1+N)} - KL = 0 \qquad \frac{1}{4} \cdot CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1+N)} - CK = 0 \qquad \frac{1}{4} \cdot CE \cdot \frac{\left(1 - 2 \cdot N + N^2\right)}{(1+N)} - ST = 0$

$2 \cdot CE \cdot \frac{N}{(1+N)} - CR = 0 \qquad CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1+N)^3} \cdot N - CF = 0$

$CE \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} \cdot N \cdot \frac{(N-1)}{(1+N)^3} - FR = 0 \qquad CE \cdot (N-1) \cdot \left(1 + 6 \cdot N + N^2\right) \cdot \frac{N}{(1+N)^4} - FQ = 0$

$\frac{1}{2} \cdot CE \cdot \frac{\left(1 + 2 \cdot N - 6 \cdot N^2 + 2 \cdot N^3 + N^4\right)}{(1+N)^3} - FL = 0$



$$GL := \frac{1}{2} \cdot \frac{CE \cdot \left(N^2 + 4 \cdot N + 1\right) \cdot (N-1)^2}{\left[N^3 + 3 \cdot N^2 + 2\ N \cdot \sqrt{(N+3) \cdot (3 \cdot N + 1)} + 3 \cdot N + 1\right]}$$

From Segment DF And HM 052201:

$$\frac{1}{2} \cdot \frac{CE \cdot \left(N^3 + 3 \cdot N^2 + 3 \cdot N - 2\ N \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} + 1\right)}{\left(N^2 + 4 \cdot N + 1\right)} - GL = 0$$

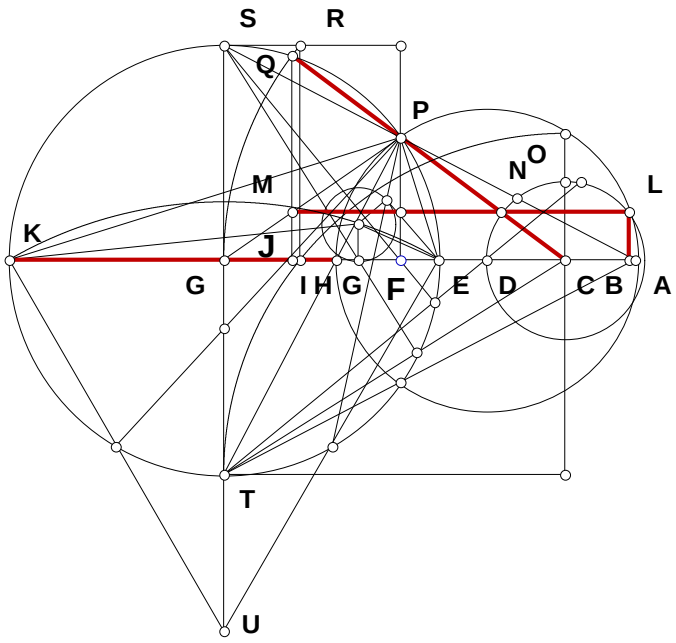
$$\frac{\frac{1}{2} \cdot \frac{CE \cdot \left(N^3 + 3 \cdot N^2 + 3 \cdot N - 2\ N \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} + 1\right)}{\left(N^2 + 4 \cdot N + 1\right)}}{\frac{1}{2} \cdot \frac{CE \cdot \left(N^2 + 4 \cdot N + 1\right) \cdot (N-1)^2}{\left[N^3 + 3 \cdot N^2 + 2\ N \cdot \sqrt{(N+3) \cdot (3 \cdot N + 1)} + 3 \cdot N + 1\right]}} = 1$$

Which does reduce to,

$$1 = 1$$

What is CD and BL?

0530011



$N_1 := 1 \qquad N_2 := 5$

$CE := N_1 \qquad EK := N_2$

$EG := \frac{EK}{2} \qquad CG := CE + EG$

$CQ := CG \qquad PQ := \frac{EG^2}{CQ}$

$CP := CQ - PQ \qquad GT := EG$

$CF := \frac{CP^2 + CG^2 - EG^2}{2 \cdot CG}$

$PF := \sqrt{CP^2 - CF^2} \qquad GS := EG$

$FG := CG - CF \qquad FS := \sqrt{FG^2 + GS^2}$

$DG := \frac{GS \cdot GT}{FG} \qquad CD := CG - DG$

$BL := \frac{PF \cdot CD}{CP}$

Some Algebraic Names;

$$CE := N_1 \qquad EK := N_2 \qquad EG - \frac{N_2}{2} = 0 \qquad CG - \left(N_1 + \frac{N_2}{2}\right) = 0$$

$$PQ - \frac{1}{2} \cdot \frac{{N_2}^2}{\left(2 \cdot N_1 + N_2\right)} = 0 \qquad CP - 2 \cdot N_1 \cdot \frac{\left(N_1 + N_2\right)}{\left(2 \cdot N_1 + N_2\right)} = 0$$

$$CF := N_1 \cdot \left(N_1 + N_2\right) \cdot \frac{\left(8 \cdot {N_1}^2 + 8 \cdot N_1 \cdot N_2 + {N_2}^2\right)}{\left(2 \cdot N_1 + N_2\right)^3}$$

$$PF - \left(N_1 + N_2\right) \cdot N_1 \cdot \sqrt{\left(N_2 + 4 \cdot N_1\right) \cdot \left(3 \cdot N_2 + 4 \cdot N_1\right)} \cdot \frac{N_2}{\left(2 \cdot N_1 + N_2\right)^3} = 0$$

$$FG - \frac{1}{2} \cdot {N_2}^2 \cdot \frac{\left(6 \cdot {N_1}^2 + 6 \cdot N_1 \cdot N_2 + {N_2}^2\right)}{\left(2 \cdot N_1 + N_2\right)^3} = 0$$

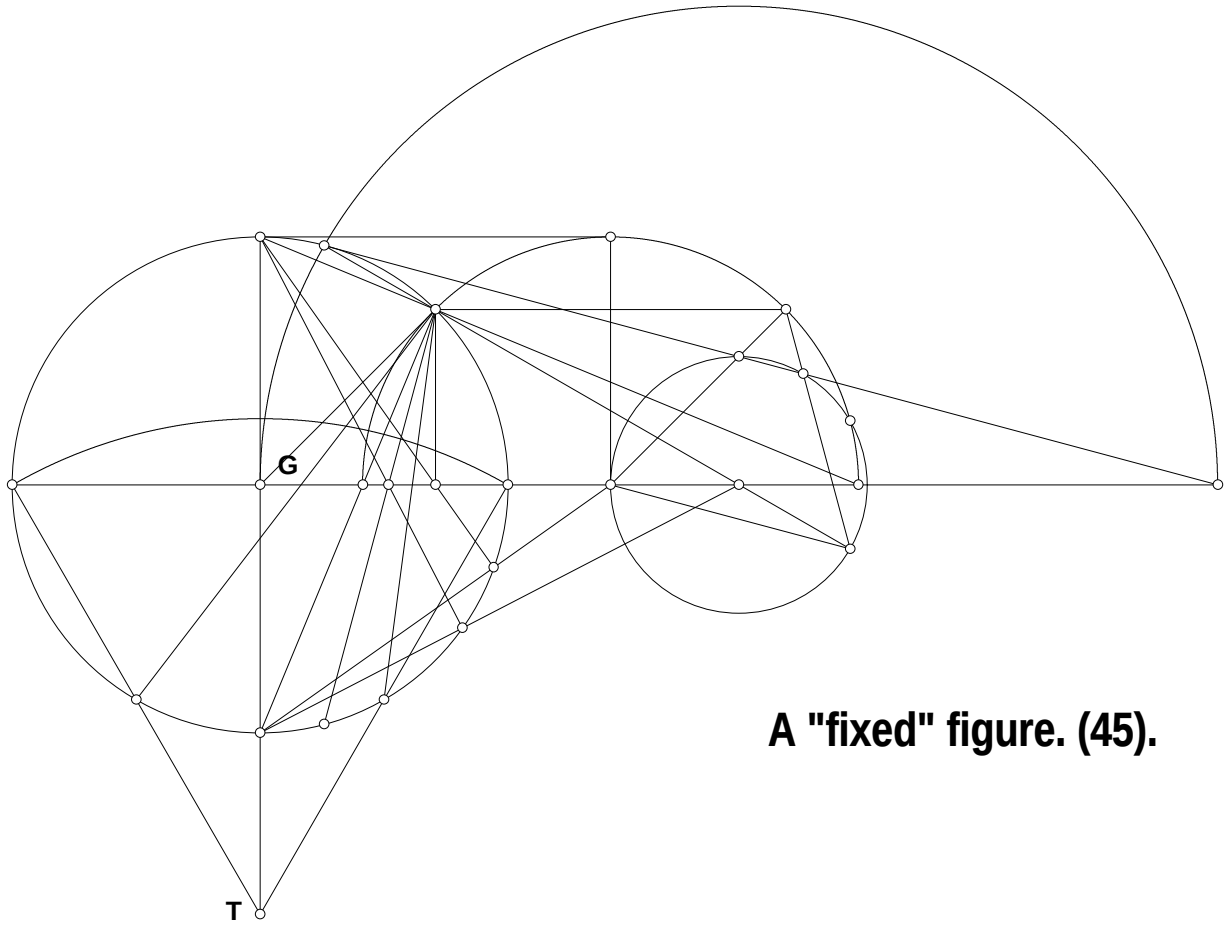
$$FS - \sqrt{\left[\frac{1}{2}\cdot N_2{}^2\cdot\frac{\left(6\cdot N_1{}^2+6\cdot N_1\cdot N_2+N_2{}^2\right)}{\left(2\cdot N_1+N_2\right)^3}\right]^2+\left(\frac{N_2}{2}\right)^2}=0 \qquad \text{OR}$$

$$FS - \left(\frac{1}{2}\right)\cdot N_2\cdot\sqrt{2}\cdot\frac{\sqrt{\left(138\cdot N_2{}^2\cdot N_1{}^4+116\cdot N_2{}^3\cdot N_1{}^3+54\cdot N_2{}^4\cdot N_1{}^2+12\cdot N_2{}^5\cdot N_1\right)\ldots}}{+\ N_2{}^6+32\cdot N_1{}^6+96\cdot N_1{}^5\cdot N_2}{\left(2\cdot N_1+N_2\right)^3}=0$$

$$DG - \frac{\left(2\cdot N_1+N_2\right)^3}{\left[2\cdot\left(6\cdot N_1{}^2+6\cdot N_1\cdot N_2+N_2{}^2\right)\right]}=0$$

$$CD - N_1\cdot\left(N_1+N_2\right)\cdot\frac{\left(2\cdot N_1+N_2\right)}{\left(6\cdot N_1{}^2+6\cdot N_1\cdot N_2+N_2{}^2\right)}=0$$

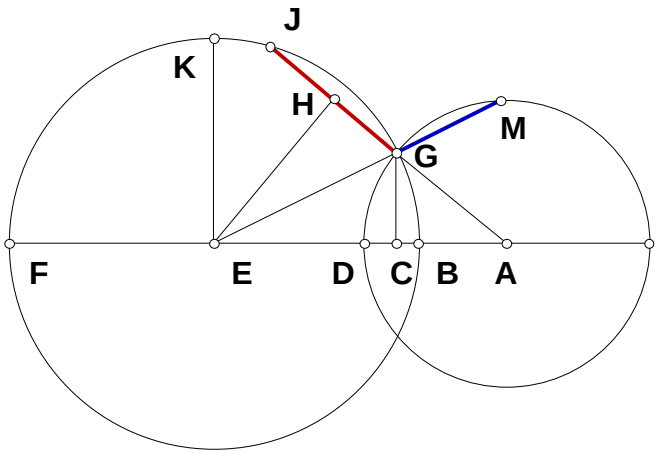
$$BL - \frac{\left(N_1+N_2\right)\cdot N_1\cdot\sqrt{\left(N_2+4\cdot N_1\right)\cdot\left(3\cdot N_2+4\cdot N_1\right)}\cdot N_2}{2\cdot\left(2\cdot N_1+N_2\right)\cdot\left(6\cdot N_1{}^2+6\cdot N_1\cdot N_2+N_2{}^2\right)}=0$$



A "fixed" figure. (45).

A Small Extrapolation 060101

Given AE, AG, and EG, what is the Algebraic name of the segment GJ?



$S_1 := 3 \quad S_2 := 2 \quad S_3 := 1$

$AE := S_1 \quad AG := S_2 \quad EG := S_3$

$$AC := \frac{AG^2 + AE^2 - EG^2}{2\,AE}$$

$$AH := \frac{AC \cdot AE}{AG} \quad GH := AH - AG$$

$HJ := GH \quad GJ := GH + HJ$

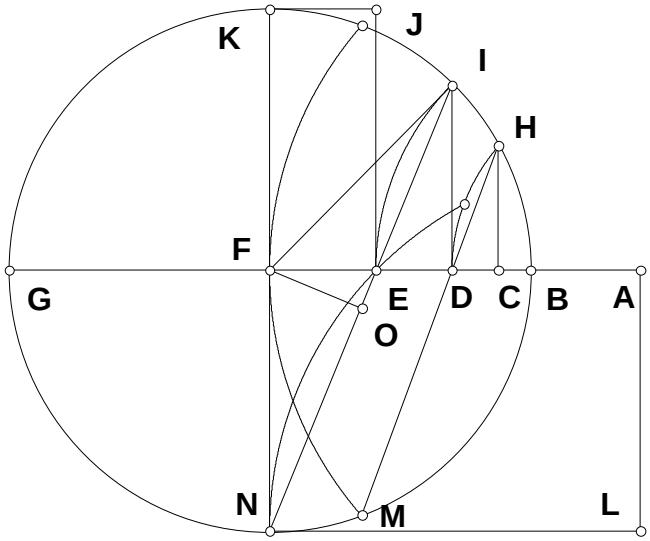
Some Algebraic Names:

$$\frac{S_2^2 + S_1^2 - S_3^2}{2\,S_1} - AC = 0 \qquad \frac{S_1^2 + S_2^2 - S_3^2}{2\,S_2} - AH = 0 \qquad \frac{S_1^2 - S_2^2 - S_3^2}{2\,S_2} - GH = 0$$

$$\frac{S_1^2 - S_2^2 - S_3^2}{S_2} - GJ = 0 \qquad \frac{S_1^2 - S_2^2 - S_3^2}{S_3} - GM = \;?$$

Units From Both Sides 060201

Start with AB as unit and find. . . . then start with . . . . as unit and find AB.



$N := 5.727 \qquad AB := .573 \qquad AG := AB \cdot N$

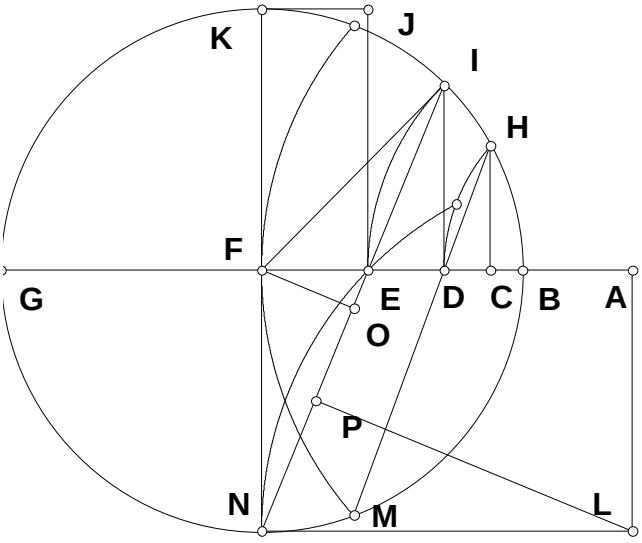
$BG := AG - AB \qquad BF := \frac{BG}{2} \qquad AF := AB + BF$

$AE := \sqrt{AB \cdot AG} \qquad BE := AE - AB$

$AB \cdot (N - 1) - BG = 0$

$\frac{1}{2} \cdot AB \cdot (N - 1) - BF = 0$

$\frac{1}{2} \cdot AB \cdot (1 + N) - AF = 0 \qquad AB\sqrt{N} - AE = 0 \qquad AB \cdot \left(\sqrt{N} - 1\right) - BE = 0$



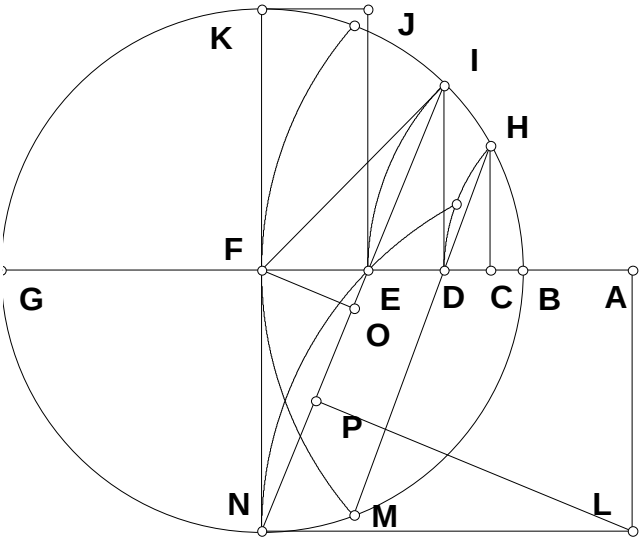
$N_2 := 2 \qquad BG_2 := 1 \qquad BF_2 := \frac{BG_2}{2}$

$BE_2 := \frac{BF_2}{N_2} \qquad EF_2 := BF_2 - BE_2$

$FN_2 := BF_2 \qquad EN_2 := \sqrt{EF_2^2 + FN_2^2}$

$NP_2 := \frac{EN_2}{2} \qquad LN_2 := \frac{EN_2 \cdot NP_2}{EF_2}$

$AF_2 := LN_2 \qquad AB_2 := AF_2 - BF_2$



$$\frac{BG_2}{2} - BF_2 = 0 \qquad \frac{BG_2}{(2 \cdot N_2)} - BE_2 = 0$$

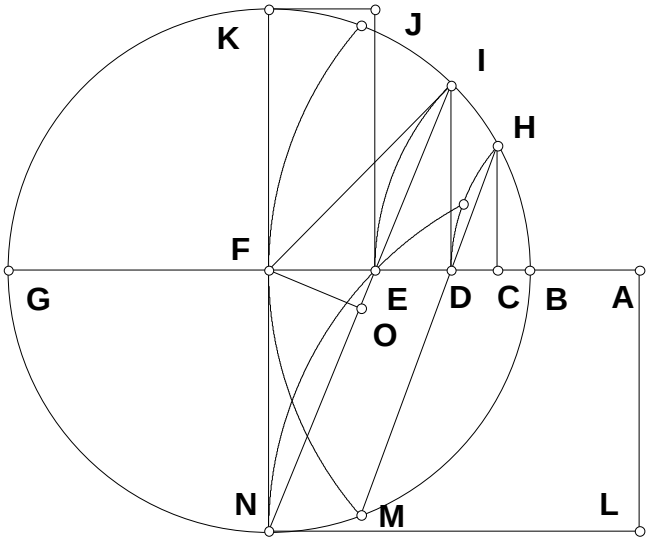
$$\frac{BG_2}{2} \cdot \frac{(N_2 - 1)}{N_2} - EF_2 = 0$$

$$\frac{BG_2}{2} \cdot \frac{\sqrt{2 \cdot N_2^2 - 2 \cdot N_2 + 1}}{N_2} - EN_2 = 0$$

$$\frac{BG_2}{4} \cdot \frac{\sqrt{2 \cdot N_2^2 - 2 \cdot N_2 + 1}}{N_2} - NP_2 = 0$$

$$\frac{BG_2}{4} \cdot \frac{(2 \cdot N_2^2 - 2 \cdot N_2 + 1)}{[N_2 \cdot (N_2 - 1)]} - LN_2 = 0$$

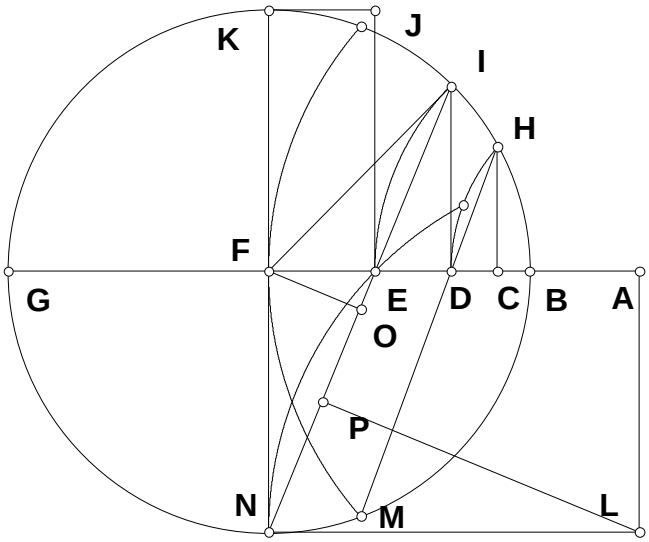
$$\frac{BG_2}{4 \cdot N_2 \cdot (N_2 - 1)} - AB_2 = 0$$



$$AI := AE \quad FI := BF \quad AD := \frac{AI^2 + AF^2 - FI^2}{2\,AF}$$

$$BD := AD - AB$$

$$AD := 2 \cdot AB \cdot \frac{N}{(N+1)} \qquad BD := AB \cdot \frac{(N-1)}{(1+N)}$$



$N_3 := 3.364 \quad BG_3 := 2.708 \quad BF_3 := \frac{BG_3}{2}$

$BD_3 := \frac{BF_3}{N_3} \quad DG_3 := BG_3 - BD_3$

$DI_3 := \sqrt{BD_3 \cdot DG_3} \quad DF_3 := BF_3 - BD_3$

$FN_3 := BF_3 \quad EF_3 := \frac{DF_3 \cdot FN_3}{FN_3 + DI_3}$

$EN_3 := \sqrt{EF_3^2 + FN_3^2} \quad NP_3 := \frac{EN_3}{2}$

$LN_3 := \frac{EN_3 \cdot NP_3}{EF_3} \quad AF_3 := LN_3$

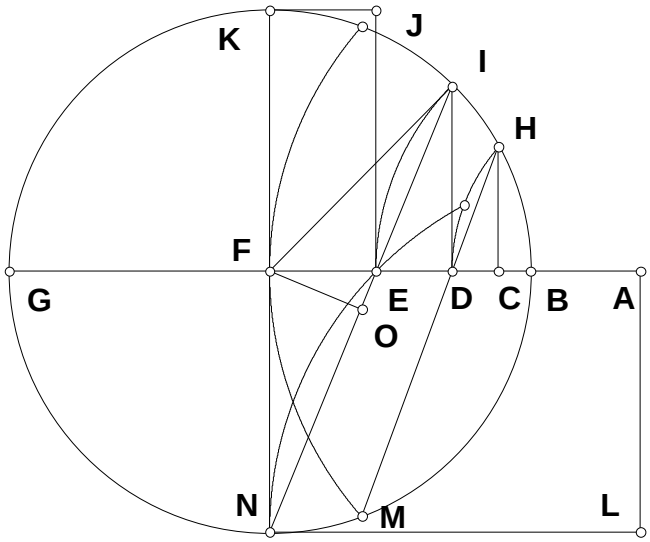
$AB_3 := AF_3 - BF_3 \quad AB_3 = 0.573$

$BD_3 := \frac{1}{2} \cdot \frac{BG_3}{N_3} \quad DG_3 := \frac{1}{2} \cdot BG_3 \cdot \frac{(2 \cdot N_3 - 1)}{N_3} \quad DI_3 := \frac{1}{(2 \cdot N_3)} \cdot BG_3 \cdot \sqrt{2 \cdot N_3 - 1}$

$DF_3 := \frac{1}{2} \cdot BG_3 \cdot \frac{(N_3 - 1)}{N_3} \quad EF_3 := \frac{1}{2} \cdot BG_3 \cdot \frac{(N_3 - 1)}{(N_3 + \sqrt{2 \cdot N_3 - 1})}$

$EN_3 := \frac{1}{2} \cdot BG_3 \cdot \sqrt{2} \cdot \sqrt{\frac{N_3}{(N_3 + \sqrt{2 \cdot N_3 - 1})}} \quad NP_3 := \frac{1}{4} \cdot BG_3 \cdot \sqrt{2} \cdot \sqrt{\frac{N_3}{(N_3 + \sqrt{2 \cdot N_3 - 1})}}$

$LN_3 := \frac{1}{2} \cdot BG_3 \cdot \frac{N_3}{(N_3 - 1)} \quad AB_3 := \frac{1}{2} \cdot \frac{BG_3}{(N_3 - 1)}$



$AH := AD$

$FH := BF \quad AC := \frac{AH^2 + AF^2 - FH^2}{2 \cdot AF}$

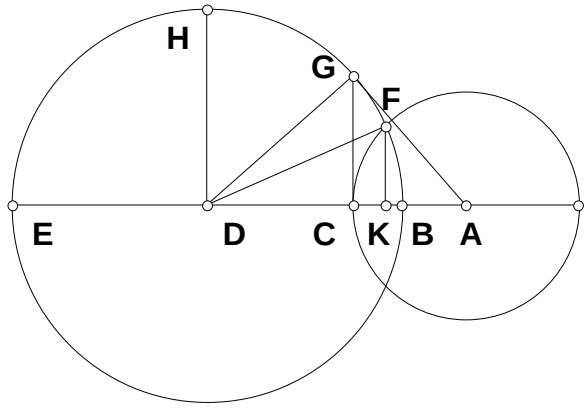
$BC := AC - AB$

$AB \cdot \left(\sqrt{N} - 1\right) - BE = 0$

$AC := AB \cdot N \cdot \frac{\left(6 \cdot N + 1 + N^2\right)}{(N+1)^3} \qquad BC := AB \cdot (3 \cdot N + 1) \cdot \frac{(N-1)}{(N+1)^3}$

Isolating A Problem 060301

If one is given point F, then finding point G would lead straightway to the solution. How is BK related to BC?



$N := 4 \qquad BE := 1$

$BD := \frac{BE}{2} \qquad BC := \frac{BE}{N} \qquad CE := BE - BC$

$CG := \sqrt{BC \cdot CE} \qquad CD := BD - BC \qquad AC := \frac{CG^2}{CD}$

$AF := AC \qquad AD := AC + CD \qquad DF := BD$

$DK := \frac{DF^2 + AD^2 - AF^2}{2\ AD} \qquad BK := BD - DK$

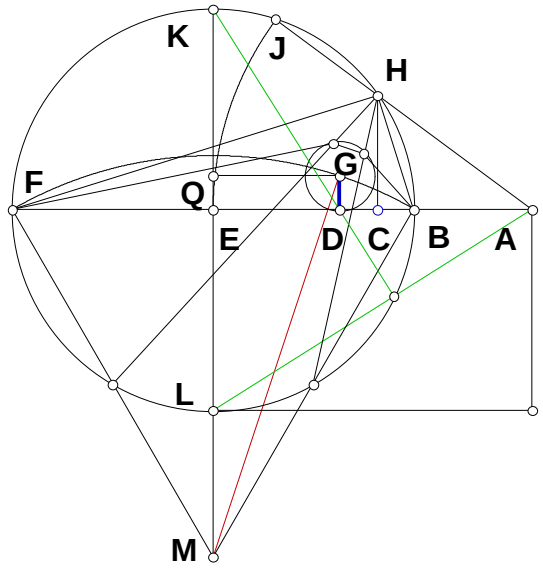
$CK := BC - BK$

$BE \cdot \frac{(N-1)}{N} - CE = 0 \qquad BE \cdot \frac{\sqrt{(N-1)}}{N} - CG = 0 \qquad BE \cdot \frac{(N-2)}{2 \cdot N} - CD = 0$

$BE \cdot \frac{2 \cdot (N-1)}{N \cdot (N-2)} - AC = 0 \qquad BE \cdot \frac{N}{2 \cdot (N-2)} - AD = 0 \qquad BE \cdot \frac{(N-2) \cdot \left(N^2 + 2 \cdot N - 2\right)}{2 \cdot N^3} - DK = 0$

$BE \cdot \frac{(3 \cdot N - 2)}{N^3} - BK = 0 \qquad BE \cdot \frac{(N-1) \cdot (N-2)}{N^3} - CK = 0 \qquad \frac{BK}{BC} - \frac{(3 \cdot N - 2)}{N^2} = 0$

For Any AB 061001



For any AB, AF what is DG?

$N := 4.39 \quad AB := .615 \quad AF := AB \cdot N$

$BF := AF - AB \quad BE := \frac{BF}{2} \quad EK := BE$

$AE := AB + BE \quad DE := \frac{EK^2}{AE} \quad EF := BE$

$FM := BF \quad EM := \sqrt{FM^2 - EF^2} \quad GM := FM$

$GQ := DE \quad MQ := \sqrt{GM^2 - GQ^2} \quad EQ := MQ - EM$

$DG := EQ$

Some Algebraic Names:

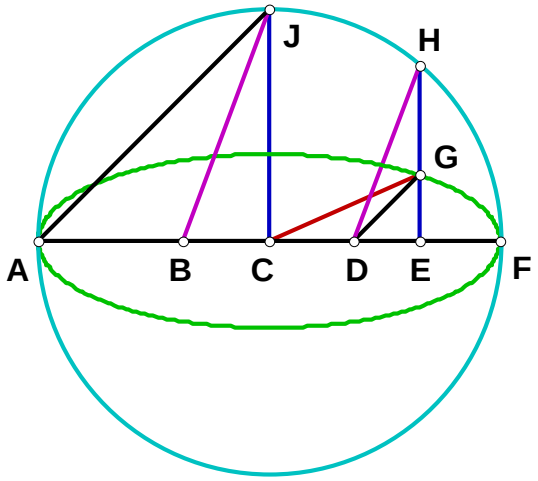
$AB \cdot (N - 1) - BF = 0 \qquad \frac{AB \cdot (N - 1)}{2} - BE = 0 \qquad \frac{1}{2} \cdot AB \cdot (N + 1) - AE = 0$

$\frac{1}{2} \cdot AB \cdot \frac{(N - 1)^2}{(N + 1)} - DE = 0 \qquad \frac{1}{2} \cdot \sqrt{3} \cdot AB \cdot (N - 1) - EM = 0$

$$\frac{AB \cdot \left[\sqrt{(N + 3) \cdot (3 \cdot N + 1)} \cdot (N - 1)\right]}{2 \cdot (N + 1)} - MQ = 0$$

$$\frac{AB \cdot (N - 1) \cdot \left[\sqrt{(N + 3) \cdot (3 \cdot N + 1)} - \sqrt{3} - \sqrt{3} \cdot N\right]}{2 \cdot (N + 1)} - DG = 0$$

Elipse By Parallels 082601



$N_1 := 4 \quad N_2 := 1$

$AF := 1 \quad AC := \frac{AF}{2} \quad CJ := AC$

$BC := \frac{AC}{N_1} \qquad AE := \frac{AF}{N_2}$

$EF := AF - AE \quad EH := \sqrt{AE \cdot EF} \quad EG := \frac{BC \cdot EH}{CJ}$

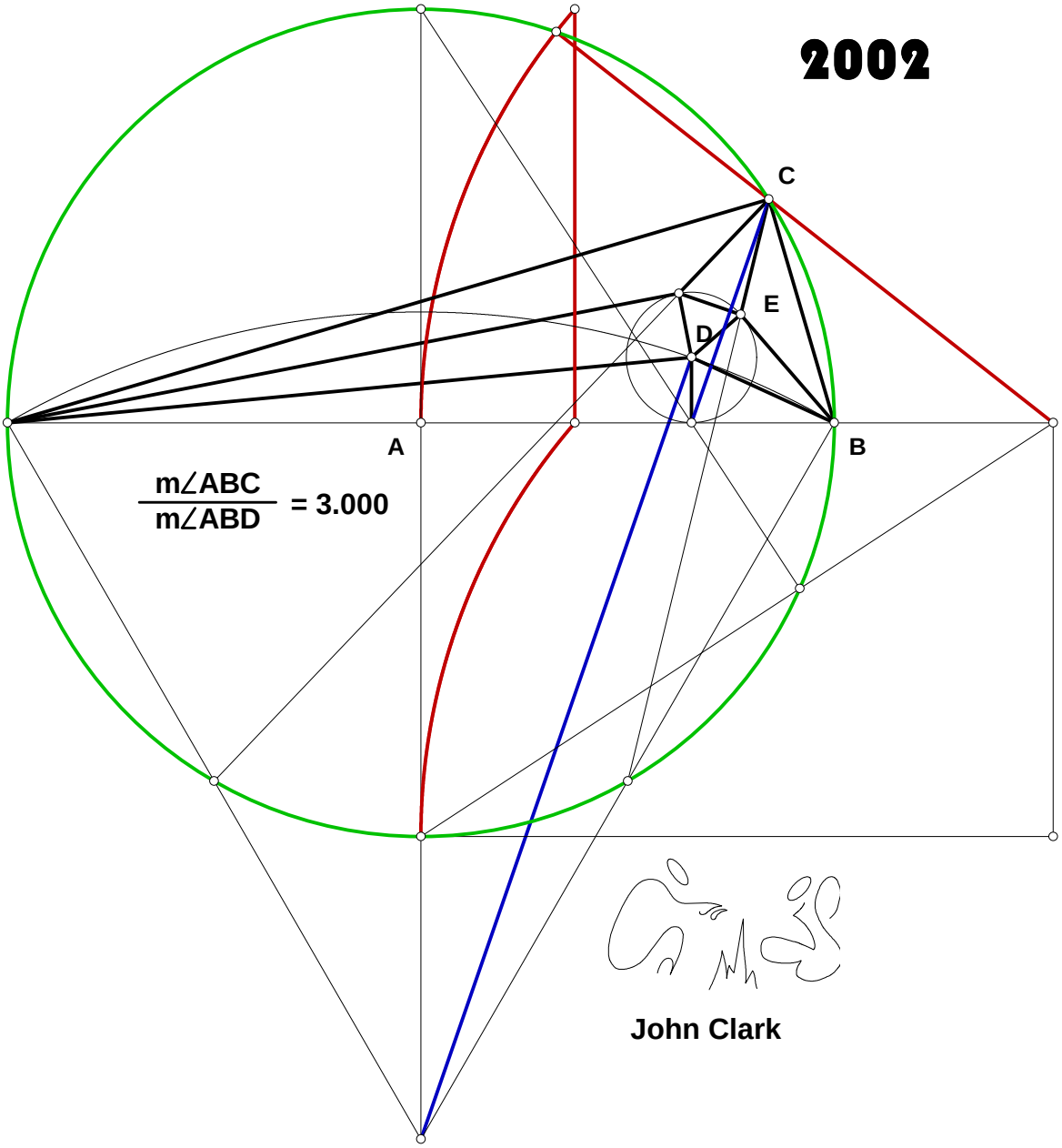
$CE := AE - AC \quad CG := \sqrt{CE^2 + EG^2}$

Some Algebraic Names:

$BC := \frac{1}{(2 \cdot N_1)} \quad AE := \frac{1}{N_2} \quad EF := 1 - \frac{1}{N_2} \quad EH := \frac{\sqrt{N_2 - 1}}{N_2} \quad EG := \frac{\sqrt{N_2 - 1}}{N_1 \cdot N_2}$

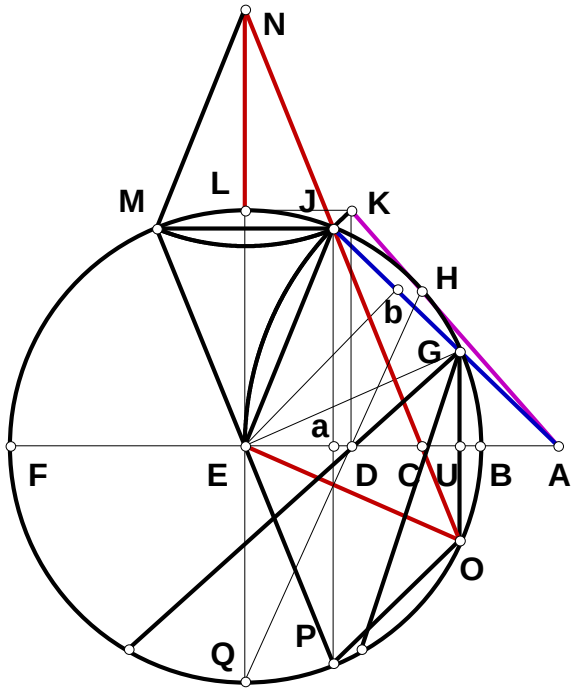
$CE := \frac{1}{N_2} - \frac{1}{2} \quad CG := \frac{1}{2} \cdot \frac{\sqrt{4 \cdot N_1^2 - 4 \cdot N_1^2 \cdot N_2 + N_1^2 \cdot N_2^2 + 4 \cdot N_2 - 4}}{(N_1 \cdot N_2)}$

Three Pieces Of Paper



John Clark

Just Another Proof Of Paper 010202



$N := 7 \quad AB := 1 \quad AF := AB \cdot N$

$BF := AF - AB \quad BE := \frac{BF}{2} \quad AE := BE + AB$

$AJ := AE \quad EJ := BE \quad Ea := \frac{EJ^2 + AE^2 - AJ^2}{2 \cdot AE}$

$Gb := Ea \quad GJ := 2 \cdot Gb \quad AG := AJ - GJ$

$Aa := AE - Ea \quad AU := \frac{Aa \cdot AG}{AJ}$

$Ja := \sqrt{AJ^2 - Aa^2} \quad GU := \frac{Ja \cdot AG}{AJ}$

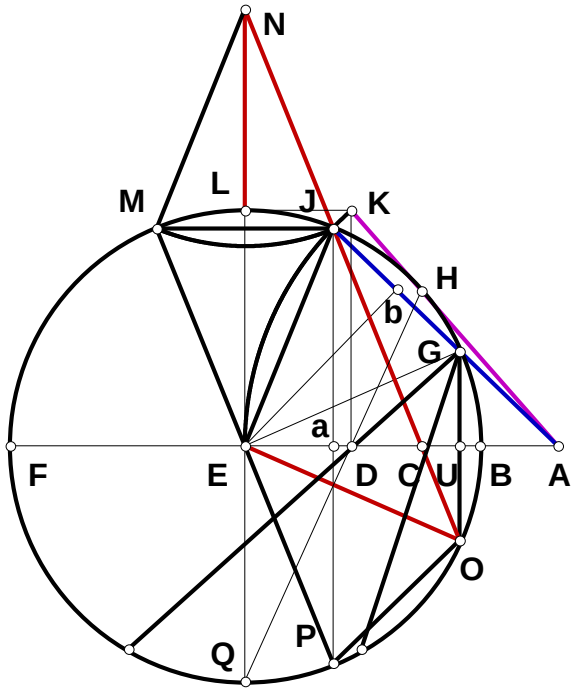
$Ua := Aa - AU \quad JO := \sqrt{Ua^2 + (GU + Ja)^2}$

$JN := \frac{JO \cdot Ea}{Ua} \quad JN - BE = 0$

From 4/29/94 $OP := \sqrt{Ja^2 - 2 \cdot Ja \cdot GU + GU^2 + Ua^2}$

$OP - 2 \cdot Ea = 0 \quad NO := JO + JN \quad EU := Ua + Ea \quad EN := \sqrt{NO^2 - EU^2} \quad EL := BE \quad LN := EN - EL$

Some Algebraic Names;



$$N - 1 - BF = 0 \qquad \frac{N-1}{2} - BE = 0$$

$$\frac{N+1}{2} - AE = 0 \qquad \frac{(N-1)^2}{4\cdot(N+1)} - Ea = 0$$

$$\frac{(N-1)^2}{2\cdot(N+1)} - GJ = 0 \qquad \frac{N^2+6\cdot N+1}{4\cdot(N+1)} - Aa = 0$$

$$\frac{2\cdot N}{N+1} - AG = 0 \qquad \frac{N\cdot\left(N^2+6\cdot N+1\right)}{(N+1)^3} - AU = 0$$

$$\frac{(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{4\cdot(N+1)} - Ja = 0$$

$$\frac{N\cdot(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{(N+1)^3} - GU = 0$$

$$\frac{\left(N^2+6\cdot N+1\right)\cdot(N-1)^2}{4\cdot(N+1)^3} - Ua = 0 \qquad \frac{(N-1)\cdot\left(N^2+6\cdot N+1\right)}{2\cdot(N+1)^2} - JO = 0 \qquad \frac{N-1}{2} - JN = 0$$

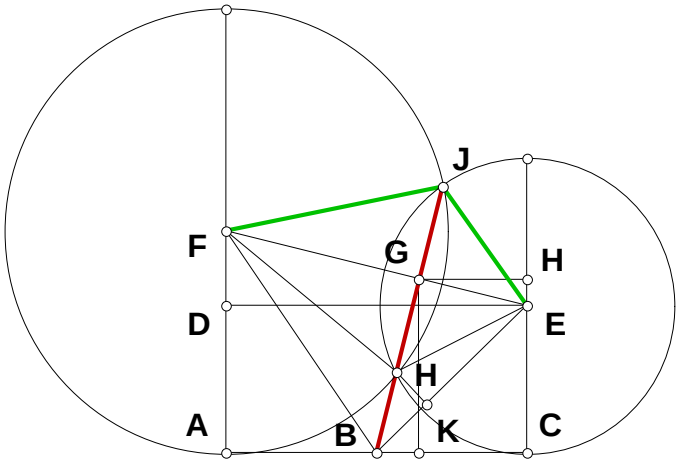
$$\frac{(N-1)^2}{2\cdot(N+1)} - OP = 0 \qquad \frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)}{(N+1)^2} - NO = 0 \qquad \frac{(N-1)^2\cdot\left(N^2+4\cdot N+1\right)}{2\cdot(N+1)^3} - EU = 0$$

$$\frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{2\cdot(N+1)^3} - EN = 0$$

$$\frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{2\cdot(N+1)^3} - \frac{N-1}{2} - LN = 0$$

Easy Power-Line 06_20_02

For any two intersecting circles, the power-line BJ intersects their common tangents AC at midpoint.



$AC := 5 \qquad AF := 4$

$CE := 3 \qquad FH := AF \quad EH := CE$

$AD := CE \quad DF := AF - AD$

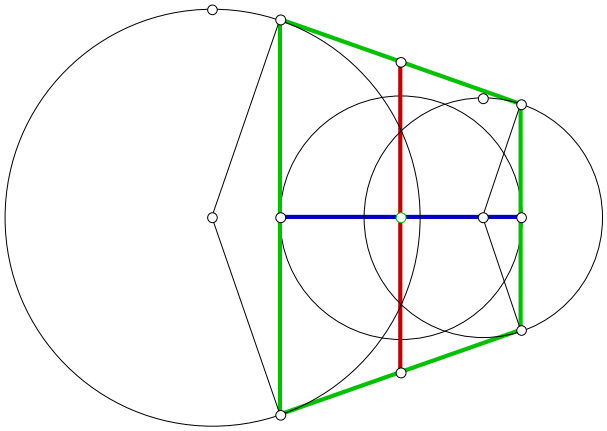
$EJ := CE \quad FJ := AF$

$DE := AC \quad EF := \sqrt{DF^2 + DE^2}$

$EG := \dfrac{EJ^2 + EF^2 - FJ^2}{2 \cdot EF}$

$EH := \dfrac{DF \cdot EG}{EF} \quad CH := CE + EH \quad GH := \dfrac{DE \cdot EG}{EF} \quad CK := GH \quad KG := CH \quad BK := \dfrac{DF \cdot KG}{DE} \quad BC := CK + BK$

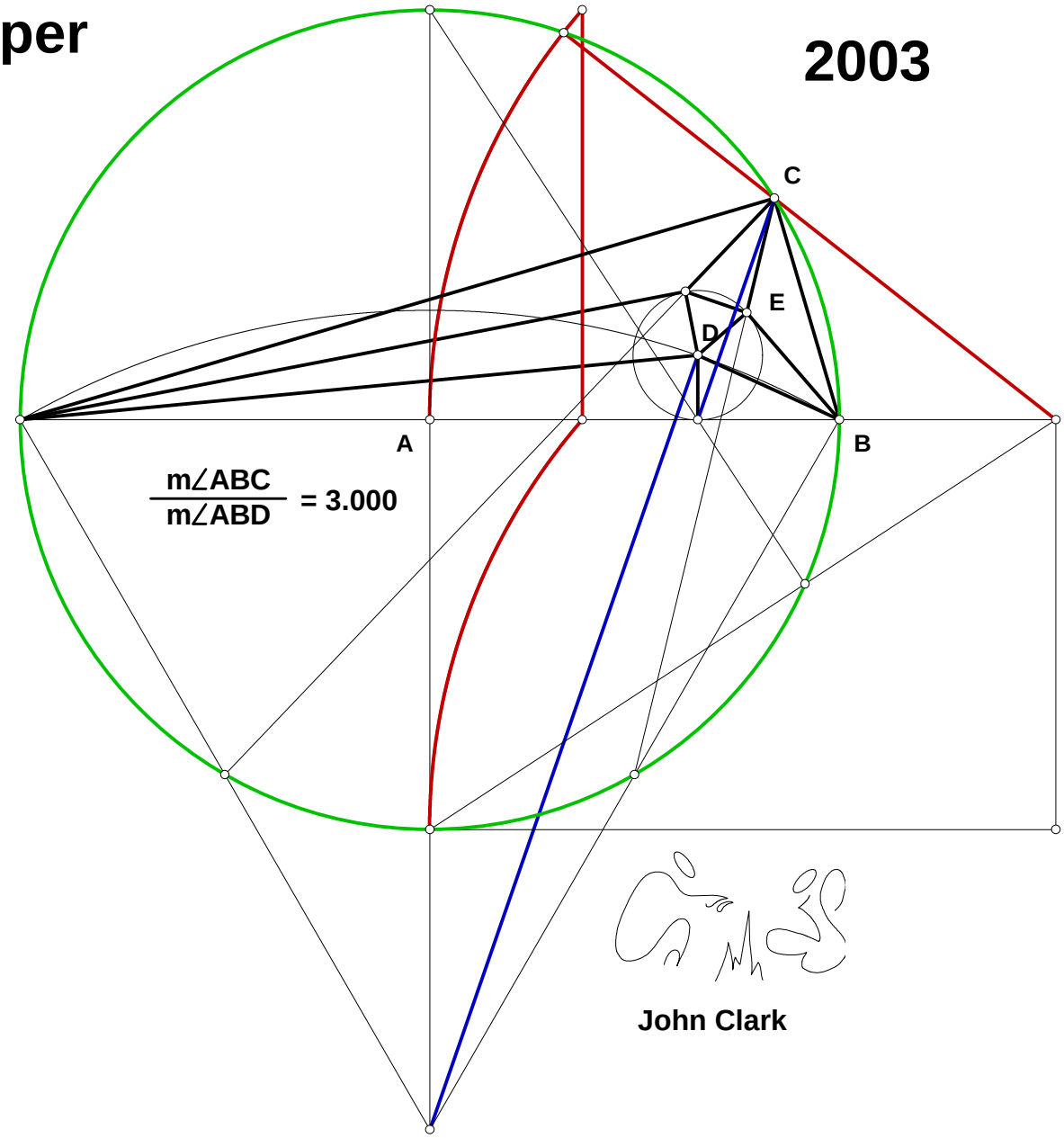
$BC - \dfrac{AC}{2} = 0$



The power line is the line upon which lays a segment between the two midpoints on the two tangents of two given circles or between a point and a circle.
The intersection of two circles lays halfway between their two tangents.
A power plane lays between two spheres or a sphere and a point.

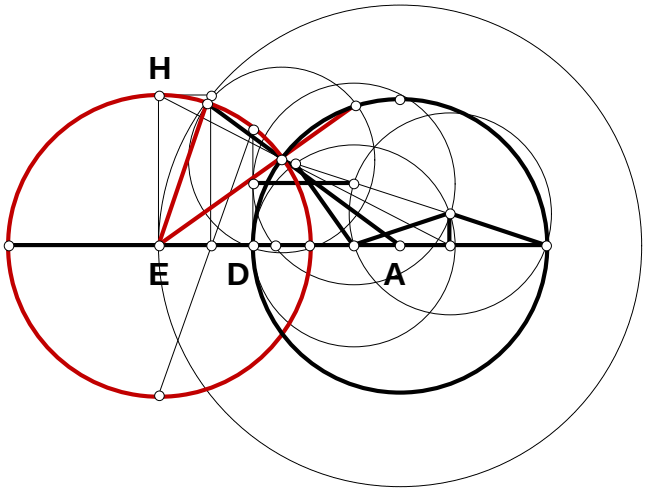
Three Pieces Of Paper

2003

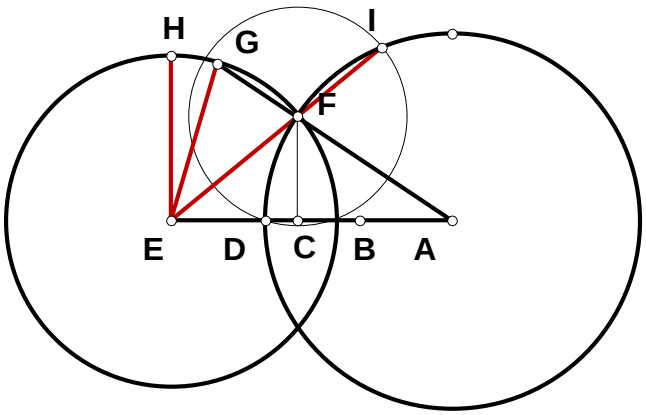


John Clark

021603



From AD project a trisection to EH.



$AD := .969 \quad AB := \frac{AD}{2} \quad N := 1.51$

$BD := AD - AB \quad BC := \frac{BD}{N}$

$CD := BD - BC \quad AC := AB + BC$

$AF := AD \quad CF := \sqrt{AF^2 - AC^2}$

$FI := \sqrt{CD^2 + CF^2}$

$DI := 2 \cdot CF \quad AI := AD$

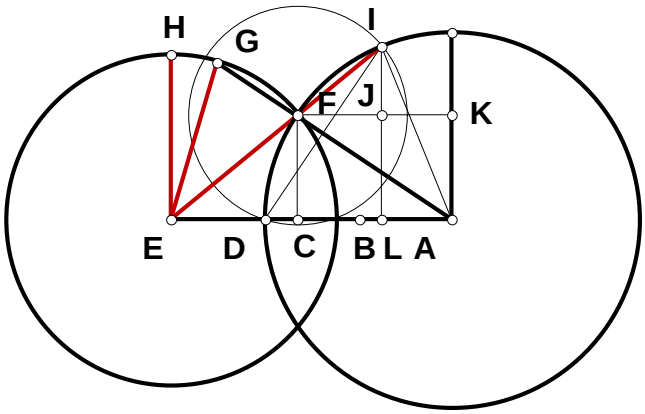
$AL := \frac{\left(2\ AD^2 - DI^2\right)}{2 \cdot AD}$

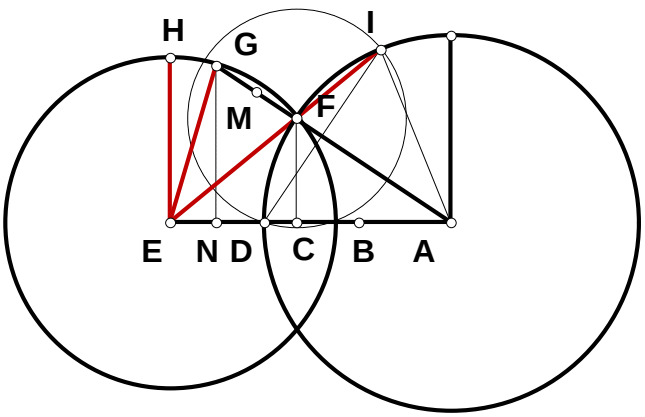
$FJ := AC - AL \quad IL := \sqrt{AI^2 - AL^2}$

$IJ := IL - CF \quad EL := \frac{FJ \cdot IL}{IJ}$

$AE := AL + EL \quad EI := \frac{FI \cdot IL}{IJ}$

$EF := EI - FI$





$$FG := \frac{EF^2 + AF^2 - AE^2}{-AF}$$

$$AG := AF + FG$$

$$AN := \frac{AC \cdot AG}{AF}$$

$$EN := AE - AN \quad FM := \frac{FG}{2}$$

$$FM - EN = 0$$

Some Algebraic Names;

$$BC - \frac{AD - AB}{N} = 0 \qquad CD - (N-1)\cdot\frac{(AD - AB)}{N} = 0 \qquad AC - \frac{(AB \cdot N + AD - AB)}{N} = 0$$

$$CF - \frac{\sqrt{(N-1)\cdot(AD - AB)\cdot(AD \cdot N + AD - AB + AB \cdot N)}}{N} = 0$$

$$FI - \sqrt{2}\cdot\sqrt{(N-1)\cdot(AD - AB)\cdot\frac{AD}{N}} = 0$$

$$DI - 2\cdot\frac{\sqrt{(N-1)\cdot(AD - AB)\cdot(AD \cdot N + AD - AB + AB \cdot N)}}{N} = 0$$

$$AL - \frac{-\left(AD^2 \cdot N^2 - 4 \cdot AD \cdot N \cdot AB + 4 \cdot AB^2 \cdot N - 2 \cdot N^2 \cdot AB^2 - 2 \cdot AD^2 + 4 \cdot AD \cdot AB - 2 \cdot AB^2\right)}{\left(AD \cdot N^2\right)} = 0$$

$$FJ - (N-1)\cdot(AD - AB)\cdot\frac{(AD \cdot N + 2 \cdot AD - 2 \cdot AB + 2 \cdot AB \cdot N)}{\left(AD \cdot N^2\right)} = 0$$

$$IL - \frac{(AB \cdot N + AD - AB)}{\left(AD \cdot N^2\right)}\cdot\sqrt{4}\cdot\sqrt{(N-1)\cdot(AD - AB)\cdot(AD \cdot N + AD - AB + AB \cdot N)} = 0$$

$$IJ - -\sqrt{(N-1)\cdot(AD - AB)\cdot(AD \cdot N + AD - AB + AB \cdot N)}\cdot\frac{(-2 \cdot AB \cdot N - 2 \cdot AD + 2 \cdot AB + AD \cdot N)}{\left(AD \cdot N^2\right)} = 0$$

$$EL - \frac{-2\cdot(N-1)\cdot(AD-AB)\cdot(AD\cdot N+2\cdot AD-2\cdot AB+2\cdot AB\cdot N)\cdot(AB\cdot N+AD-AB)}{AD\cdot\left[N^2\cdot(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)\right]} = 0$$

$$AE - -AD^2\cdot\frac{N}{(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)} = 0$$

$$EI - 2\cdot\sqrt{2}\cdot\sqrt{(N-1)\cdot(AD-AB)\cdot\frac{AD}{N}}\cdot\frac{(AB\cdot N+AD-AB)}{(2\cdot AB\cdot N+2\cdot AD-2\cdot AB-AD\cdot N)} = 0$$

$$EF - -\sqrt{2}\cdot\sqrt{(N-1)\cdot(AD-AB)\cdot\frac{AD}{N}}\cdot AD\cdot\frac{N}{(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)} = 0$$

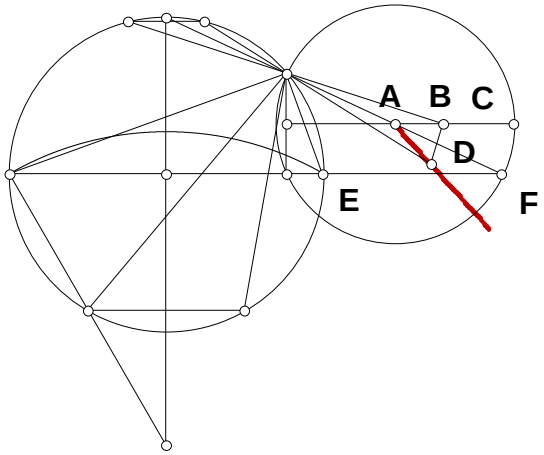
$$FG - -2\cdot AD\cdot(AD-AB)\cdot\frac{(N-1)}{(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)} = 0$$

$$AG - -AD^2\cdot\frac{N}{(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)} = 0$$

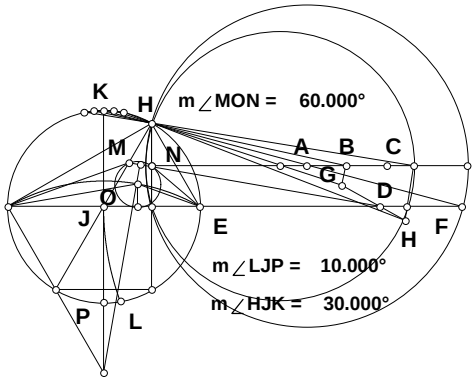
$$AN - (AB\cdot N+AD-AB)\cdot\frac{AD}{(2\cdot AB\cdot N+2\cdot AD-2\cdot AB-AD\cdot N)} = 0$$

$$EN - -AD\cdot(N-1)\cdot\frac{(AD-AB)}{(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)} = 0 \qquad FM - EN = 0$$

022803

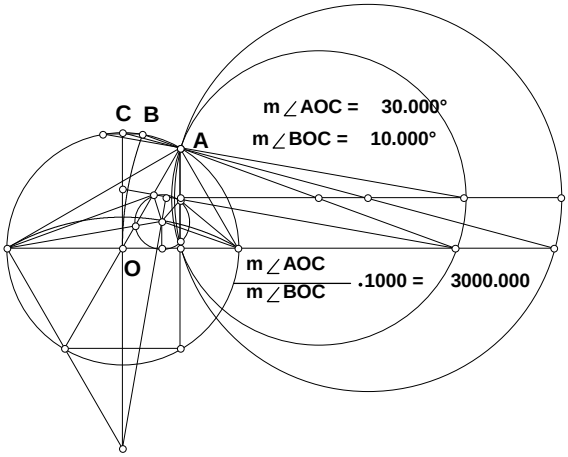


If D were between EF, then it would be the point sought for angle trisection. When is moved between A and C the locus AD is formed. This locus is not straight, but it is fairly straight.

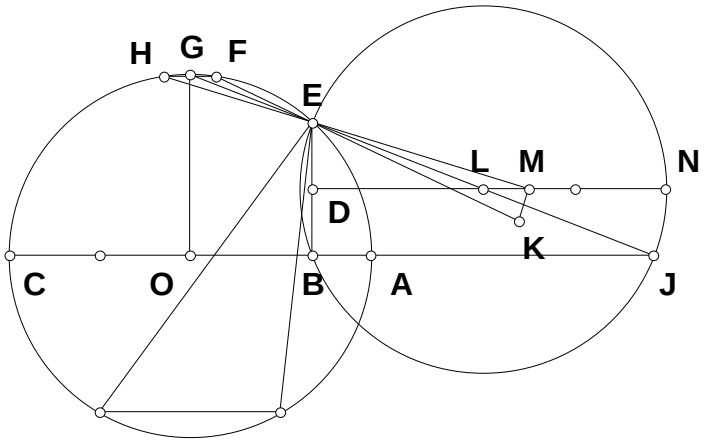


Even if a large segment is used from two points on AD, say GH for an intercept, this drawing program claims that I have attained a trisection for any angle.

If a very small segment is taken, as below, tolerance is beyond the drawing program. What does it look like using Algebra? Trisection to within millionths, in some circumstances, may be tolerable.



As the different points of intersection is no actually viewable on the finer figure, the rougher figure will be used for drawings, but the equations will refer to the finer.



$N_1 := 1 \qquad N_2 := 4$

$N_3 := 2$

$AC := N_1 \quad AO := \frac{AC}{2}$

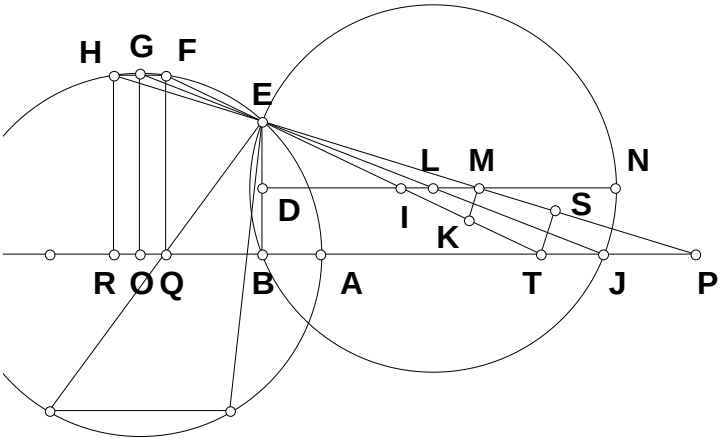
$AB := \frac{AC}{N_2} \quad BC := AC - AB$

$BE := \sqrt{AB \cdot BC} \quad DE := \frac{BE}{2}$

$BO := AO - AB \quad GO := AO$

$DL := \frac{BO \cdot DE}{GO - BE} \quad EL := \sqrt{DL^2 + DE^2}$

$LN := EL \quad LM := \frac{LN}{N_3} \quad DM := DL + LM \quad EM := \sqrt{DE^2 + DM^2} \quad EP := \frac{EM \cdot BE}{DE}$



$BP := \frac{DM \cdot BE}{DE} \quad PO := BP + BO$

$EH := \frac{AO^2 + EP^2 - PO^2}{-EP}$

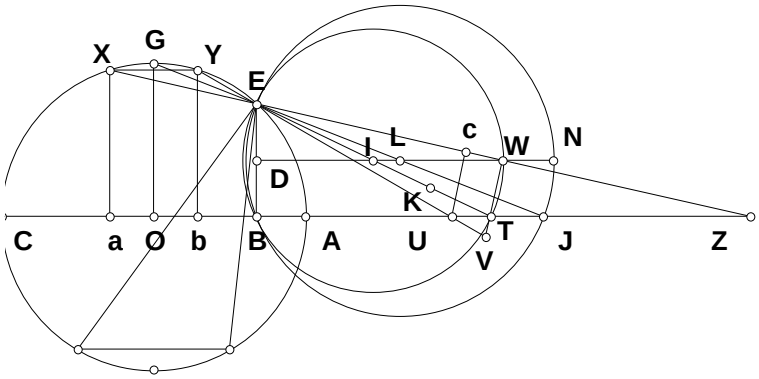
$HP := EP + EH \quad PR := \frac{BP \cdot HP}{EP}$

$OR := PR - PO \quad BQ := BO - OR$

$HR := \frac{BE \cdot HP}{EP} \quad BT := \frac{BQ \cdot BE}{HR - BE}$

$ET := \sqrt{BE^2 + BT^2} \quad PT := BP - BT \quad PS := \frac{PT^2 + EP^2 - ET^2}{2 \cdot EP} \quad ES := EP - PS \quad ST := \sqrt{ET^2 - ES^2}$

$KM := \frac{ST \cdot EM}{ES} \quad EI := \frac{ET}{2} \quad EK := \frac{ET \cdot EM}{ES} \quad OT := BT + BO$



$IW := EI \quad DI := \sqrt{EI^2 - DE^2}$

$DW := DI + IW$

$EW := \sqrt{DE^2 + DW^2}$

$EZ := \frac{EW \cdot BE}{DE} \quad BZ := \frac{DW \cdot BE}{DE}$

$OZ := BZ + BO$

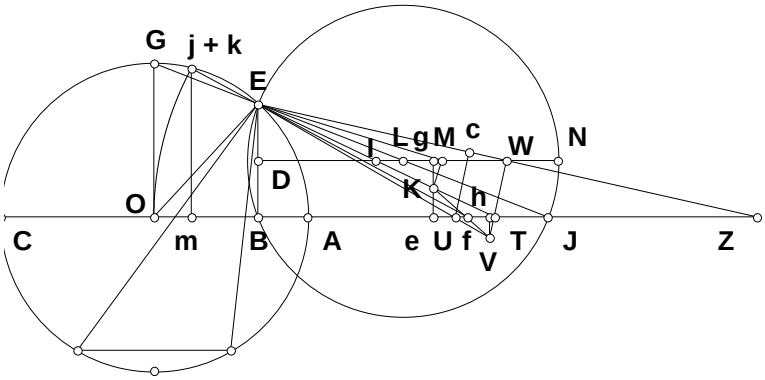
$TW := \sqrt{ET^2 - EW^2}$

$EX := \frac{AO^2 + EZ^2 - OZ^2}{-EZ} \quad XZ := EZ + EX \quad Za := \frac{BZ \cdot XZ}{EZ} \quad Oa := Za - OZ \quad Bb := BO - Oa$

$Xa := \frac{BE \cdot XZ}{EZ} \quad BU := \frac{Bb \cdot BE}{Xa - BE} \quad EU := \sqrt{BE^2 + BU^2} \quad UZ := BZ - BU \quad Zc := \frac{UZ^2 + EZ^2 - EU^2}{2 \cdot EZ}$

$Uc := \sqrt{UZ^2 - Zc^2} \quad VW := \frac{Uc \cdot EW}{EZ - Zc} \quad OU := BU + BO$

$IM := DM - DI \quad IK := EK - EI$



$Ig := \frac{IK^2 + IM^2 - KM^2}{2 \cdot IM}$

$EV := \frac{EU \cdot EW}{EZ - Zc}$

$TU := OT - OU$

$UV := EV - EU$

$TV := VW - TW$

$Uh := \frac{UV^2 + TU^2 - TV^2}{2 \cdot TU}$

$Kg := \sqrt{IK^2 - Ig^2} \quad Ke := DE - Kg \quad Vh := \sqrt{UV^2 - Uh^2} \quad Dg := DI + Ig \quad Bh := BU + Uh$

$eh := Bh - Dg \quad Ke := DE - Kg \quad ef := \frac{eh \cdot Ke}{(Ke + Vh)} \quad Bf := Dg + ef \quad Of := Bf + BO$

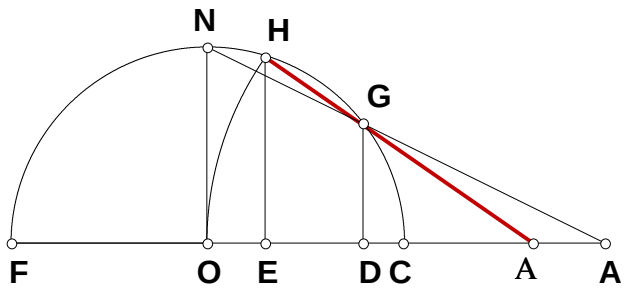
$Om := \frac{AO^2}{2 \cdot Of} \quad Ej := \frac{AO^2}{Of} \quad Om - \frac{Ej}{2} = 0 \quad Ef := \sqrt{BE^2 + Bf^2} \quad Ek := \frac{AO^2 + Ef^2 - Of^2}{-Ef}$

$Om - \frac{Ek}{2} = -0.000000000002639 \quad \frac{Ek}{Ej} = 1.000000000030396$

And so the Trisection is accurate to within a few decimal places. 30 degree angle shown.

The Gravitating Answer. 030503

How many itterations to go beyond 15 decimal place precision in trisection? The itteration is from AO where GH determines a new A.



$$\Delta := 22 \qquad \delta := 0 .. \Delta \qquad N := 2$$

$$CF := 1 \qquad CO := \frac{CF}{2}$$

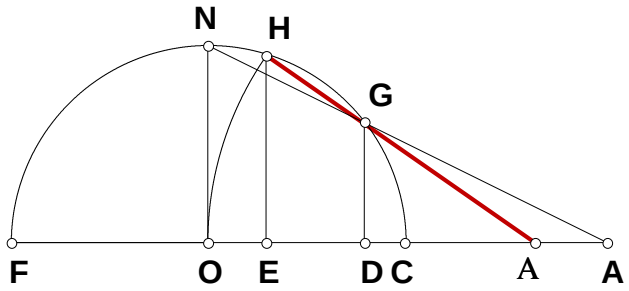
$$CD := \frac{CO}{N} \qquad DO := CO - CD$$

$$DF := DO + CO \qquad DG := \sqrt{CD \cdot DF}$$

$$AD_0 := \frac{DO \cdot DG}{CO - DG} \qquad AO_0 := AD_0 + DO \qquad EO_0 := \frac{CO^2}{2 \cdot (AD_0 + DO)} \qquad DE_0 := DO - EO_0$$

$$EH_0 := \sqrt{(CO + EO_0) \cdot (CO - EO_0)}$$

$$\begin{bmatrix} AD_{\delta+1} \\ AO_{\delta+1} \\ EO_{\delta+1} \\ DE_{\delta+1} \\ EF_{\delta+1} \\ EH_{\delta+1} \end{bmatrix} := \begin{bmatrix} \frac{DE_\delta \cdot DG}{EH_\delta - DG} \\ \frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \\ \frac{CO^2}{2 \cdot \left(\frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \\ DO - \frac{CO^2}{2 \cdot \left(\frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \\ CO + \frac{CO^2}{2 \cdot \left(\frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \\ \sqrt{\left[CO - \frac{CO^2}{2 \cdot \left(\frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \right] \cdot \left[CO + \frac{CO^2}{2 \cdot \left(\frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \right]} \end{bmatrix}$$



$$AG := \sqrt{DG^2 + \left(AD_{\Delta}\right)^2}$$

$$GH := \frac{CO^2 + AG^2 - \left(AO_{\Delta}\right)^2}{-AG}$$

$$EO_{\Delta} - \frac{GH}{2} = 0$$

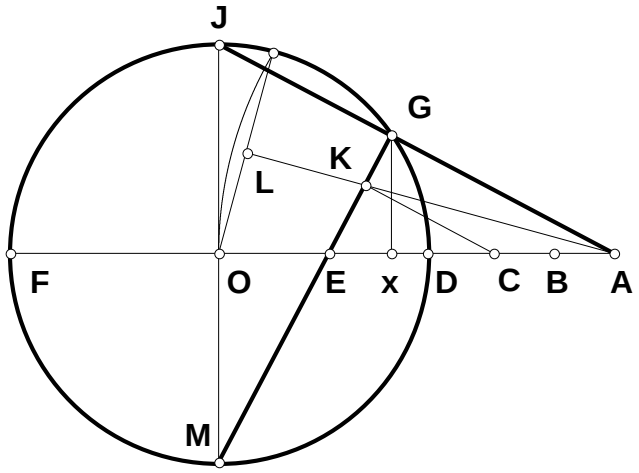
The displayed precision is for 15 decimal places. Trisection is beyond that. Since the physical world is quantitized, physical trisection is possible.

$EO_{\delta} - \frac{GH}{2}$	δ
-0.019836790725685	0
-0.004498504834066	1
-0.001019750164038	2
-0.000231158851096	3
-0.000052399460482	4
-0.000011877993414	5
-0.000002692522516	6
-0.000000610345304	7
-0.000000138354048	8
-0.000000031362317	9
-0.000000007109260	10
-0.000000001611539	11
-0.000000000365306	12
-0.000000000082808	13
-0.000000000018771	14
-0.000000000004255	15
-0.000000000000965	16
-0.000000000000219	17
-0.000000000000050	18
-0.000000000000011	19
$-2.789435349370706 \cdot 10^{-15}$	20
0	21
0	22

One can see how rapidly each recursion increases precision. And so, for any required precision, one can trisect an angle grearter than that, relatively rapidly--especially if one combine yesterdays plate with today's.

030803

Is anything saved by starting from a much more precise point for itteration demonstrated in 0305?



$DF := 1 \qquad DO := \frac{DF}{2}$

$N := 2 \qquad Dx := \frac{DO}{N}$

$Ox := DO - Dx \qquad Fx := DF - Dx$

$Gx := \sqrt{Dx \cdot Fx} \qquad AO := \frac{Ox \cdot DO}{DO - Gx}$

$HO := DO \qquad LO := \frac{HO}{2}$

$Ex := \frac{Ox \cdot Gx}{Gx + DO} \qquad Ob := \frac{LO^2}{AO}$

$Lb := \sqrt{LO^2 - Ob^2} \qquad ab := \frac{Ex \cdot Lb}{Gx}$

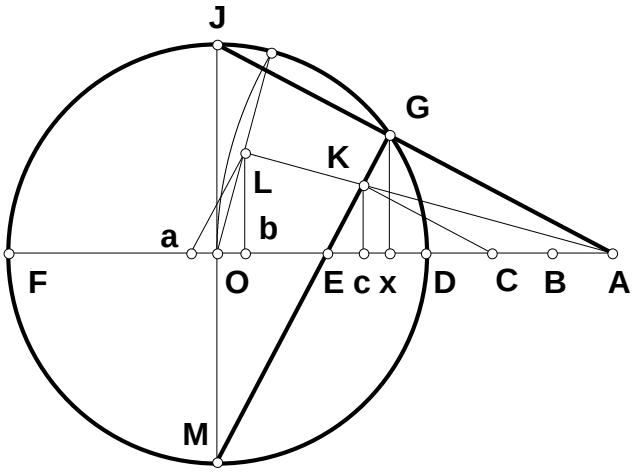
$Aa := AO + ab - Ob$

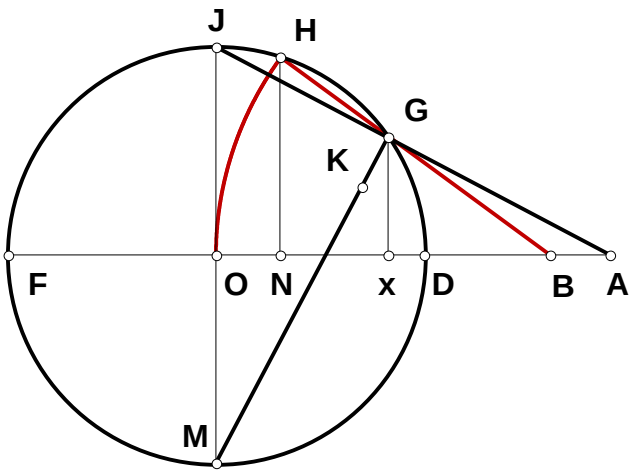
$Ax := AO - Ox \qquad AE := Ax + Ex$

$Kc := \frac{Lb \cdot AE}{Aa} \qquad Cc := \frac{AO \cdot Kc}{DO}$

$Ec := \frac{ab \cdot AE}{Aa} \qquad CE := Cc + Ec \qquad AC := AE - CE$

$AB := \frac{AC}{2}$





$$\Delta := 22 \qquad \delta := 0 .. \Delta$$

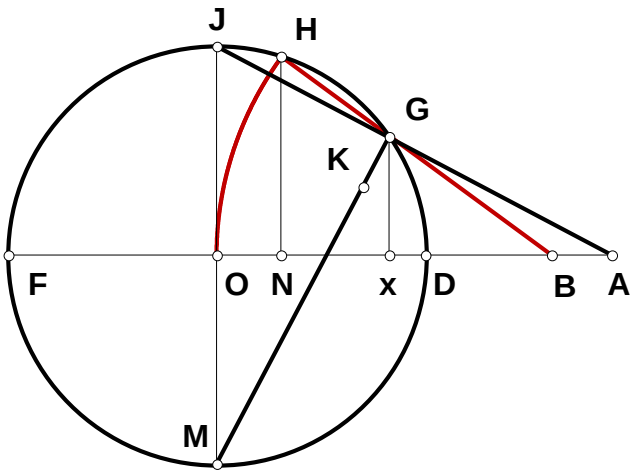
$$BO_0 := AO - AB$$

$$NO_0 := \frac{DO^2}{2\ BO_0}$$

$$Nx_0 := Ox - NO_0$$

$$HN_0 := \sqrt{\left(DO + NO_0\right) \cdot \left(DO - NO_0\right)}$$

$$\begin{bmatrix} BO_{\delta+1} \\ NO_{\delta+1} \\ Nx_{\delta+1} \\ HN_{\delta+1} \end{bmatrix} := \begin{bmatrix} \dfrac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta \\ \dfrac{DO^2}{2\left(\dfrac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta\right)} \\ Ox - \dfrac{DO^2}{2\left(\dfrac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta\right)} \\ \sqrt{\left[DO + \dfrac{DO^2}{2\left(\dfrac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta\right)}\right] \cdot \left[DO - \dfrac{DO^2}{2\left(\dfrac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta\right)}\right]} \end{bmatrix}$$



$$Bx_{\Delta} := BO_{\Delta} - Ox$$

$$BG := \sqrt{Gx^2 + \left(Bx_{\Delta}\right)^2}$$

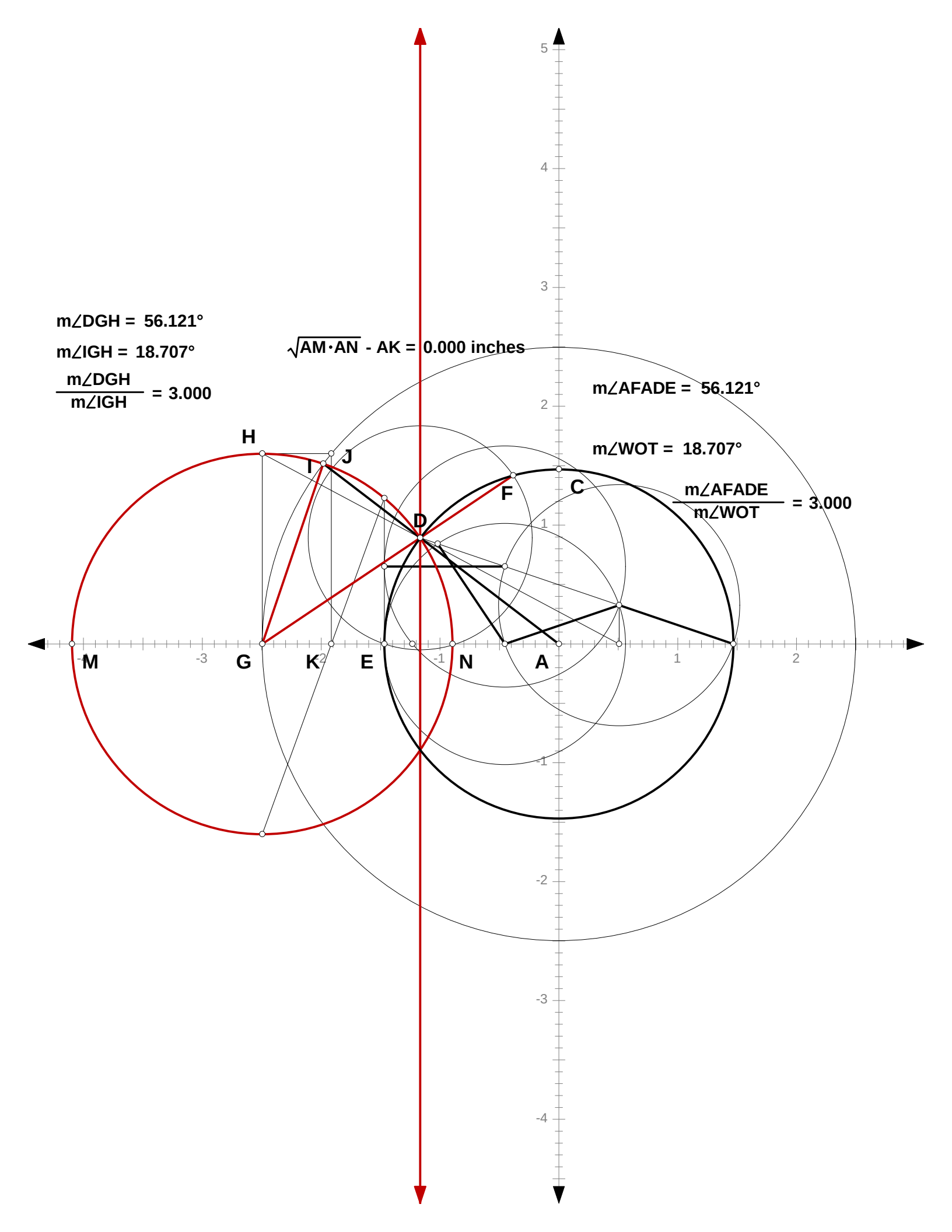
$$GH := \frac{DO^2 + BG^2 - \left(BO_{\Delta}\right)^2}{-BG}$$

$$NO_{\Delta} - \frac{GH}{2} = 0$$

$NO_{\delta} - \frac{GH}{2}$

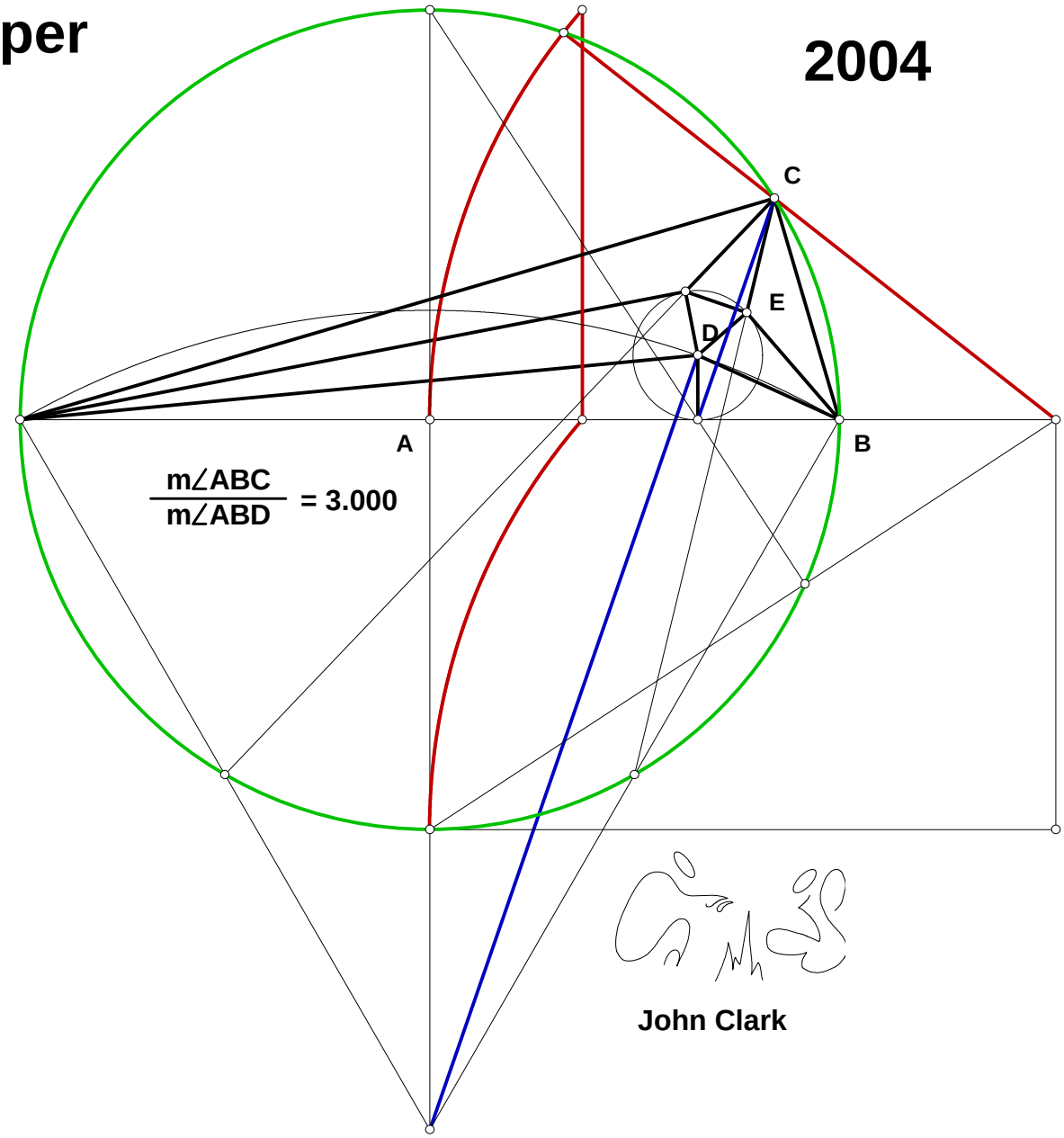
δ
-0.000733072475044	0
-0.000166174134206	1
-0.000037668619212	2
-0.000008538782769	3
-0.000001935584914	4
-0.000000438761479	5
-0.000000099459153	6
-0.000000022545559	7
-0.000000005110663	8
-0.000000001158493	9
-0.000000000262609	10
-0.000000000059529	11
-0.000000000013494	12
-0.000000000003059	13
-0.000000000000693	14
-0.000000000000157	15
-0.000000000000035	16
$-7.965850201685498 \cdot 10^{-15}$	17
$-1.706967900361178 \cdot 10^{-15}$	18
0	19
0	20
0	21
0	22

Although one starts off in a more precise spot, not much in the way of steps for 15 decimal place precision is saved. The steps are a waste of time.


m∠DGH = 56.121°
m∠IGH = 18.707°
m∠DGH / m∠IGH = 3.000
√AM·AN - AK = 0.000 inches
m∠AFADE = 56.121°
m∠WOT = 18.707°
m∠AFADE / m∠WOT = 3.000
H
J
I
F
C
D
M
G
K
E
N
A
5
4
3
2
1
-1
-2
-3
-4
-3
-2
-1
1
2

Three Pieces Of Paper

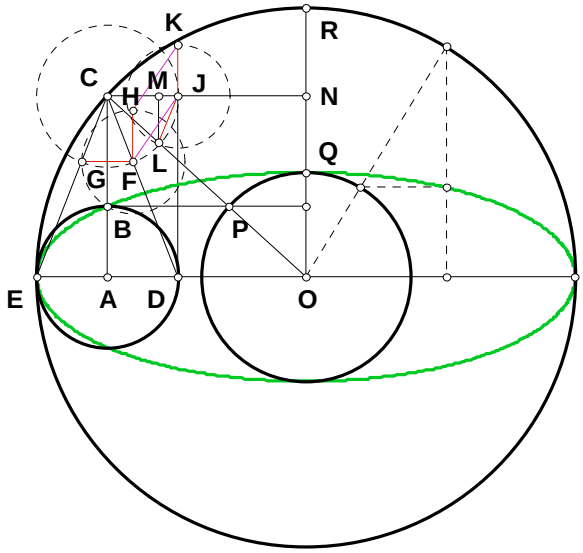
2004



John Clark

Given AE, AC find the ellipse

031704.mcd



Or given any AE, AC find the diameter of the Circle.

$N := 3$ $AE := 1$

$AC := AE \cdot N$

$DE := AE \cdot 2$ $CJ := AE$

$CE := \sqrt{AE^2 + AC^2}$

$CG := AE$ $FG := \frac{DE \cdot CG}{CE}$

$JL := FG$ $JM := \frac{JL^2}{2 \cdot CJ}$

$CM := CJ - JM$ $LM := \sqrt{JL^2 - JM^2}$ $NO := AC$ $CN := \frac{CM \cdot NO}{LM}$ $AO := CN$ $EO := AO + AE$

$OP := \frac{AE^2}{LM}$ $Major := EO$ $Minor := OP$

$Major = 5$

$Minor = 1.66667$

$Major - \frac{N^2 + 1}{2} = 0$ $Minor - \frac{N^2 + 1}{2 \cdot N} = 0$

$\frac{Major}{Minor} - N = 0$

Given AC, AB and either pont of contact, D or F from any C, what is the lenght of the cord DF cut off by a line from any C? **032304**

$N_1 := 1.708$ $N_2 := 1.649$

$N_3 := 1.24$

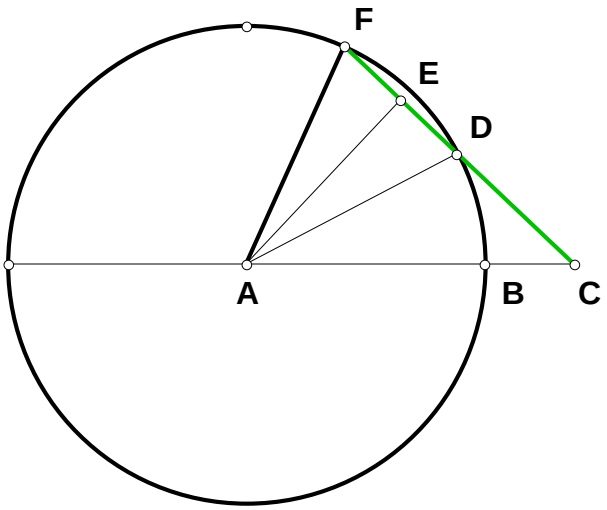
$AC := N_1$ $CF := N_2$

$AF := N_3$

$$DF_1 := \frac{N_3^2 + N_2^2 - N_1^2}{N_2}$$

$CD := CF - DF_1$ $N_2 := CD$

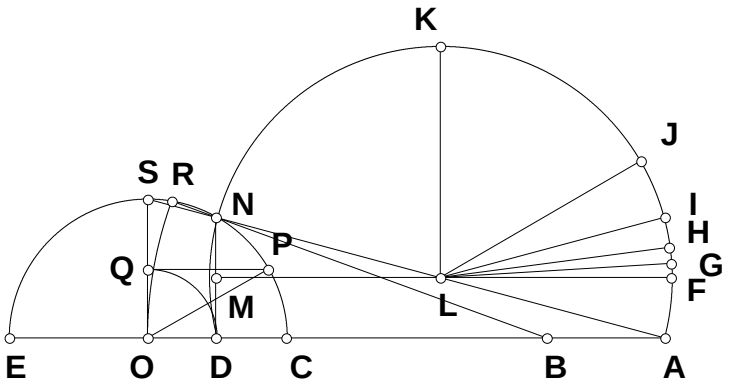
$$DF_2 := \frac{N_3^2 + N_2^2 - N_1^2}{N_2}$$



$DF_1 = 0.812$ $DF_2 = -0.812$ $DF_1 + DF_2 = 0$

0405043

All this extra work and I have lost accuracy from 022803!



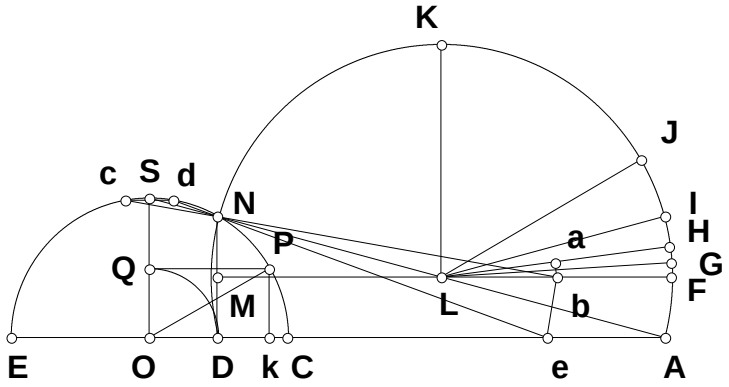
$N := 4 \qquad CE := 1$

$CO := \frac{CE}{2} \qquad CD := \frac{CE}{N}$

$DO := CO - CD$

$DN := \sqrt{(CO + DO)\cdot(CO - DO)}$

$AD := \frac{DO \cdot DN}{CO - DN} \qquad DM := \frac{DN}{2}$



$AN := \sqrt{AD^2 + DN^2} \qquad HL := \frac{AN}{2} \qquad LA := \frac{HL}{2} \qquad Ok := \sqrt{CO^2 - DO^2} \qquad Ck := CO - Ok$

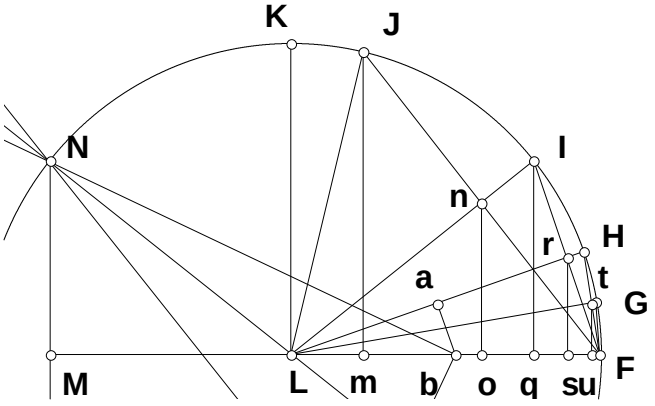
$Fm := \frac{Ck \cdot HL}{CO} \qquad Jm := \frac{DO \cdot HL}{CO}$

$JF := \sqrt{Fm^2 + Jm^2} \qquad Fn := \frac{JF}{2}$

$Fo := \frac{Fm \cdot Fn}{JF} \qquad Lo := HL - Fo$

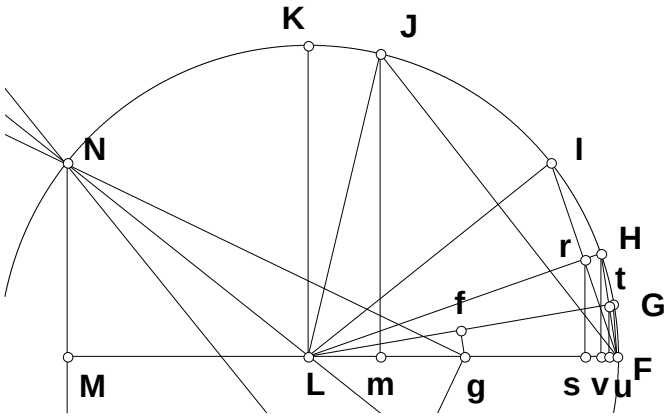
$no := \frac{Jm}{2} \qquad Ln := \sqrt{no^2 + Lo^2}$

$Iq := \frac{no \cdot HL}{Ln} \qquad Lq := \frac{Lo \cdot HL}{Ln}$



$Fq := HL - Lq \qquad FI := \sqrt{Iq^2 + Fq^2} \qquad Fr := \frac{FI}{2} \qquad Fs := \frac{Fq \cdot Fr}{FI} \qquad Ls := HL - Fs \qquad Lr := \sqrt{HL^2 - Fr^2}$

$Lb := \frac{HL^2}{2 \cdot Lr} \qquad rs := \frac{Iq}{2}$



$$Lv := \frac{Ls \cdot HL}{Lr} \quad Hv := \frac{rs \cdot HL}{Lr}$$

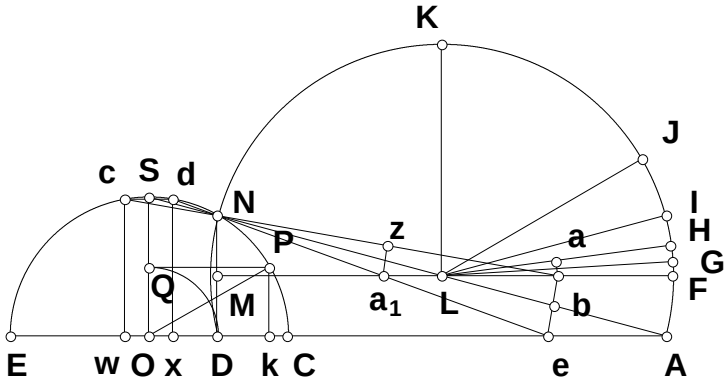
$$Lv := \frac{Ls \cdot HL}{Lr} \quad Fv := HL - Lv$$

$$FH := \sqrt{Hv^2 + Fv^2} \quad Ft := \frac{FH}{2}$$

$$Lf := \frac{HL}{2} \quad Fu := \frac{Fv \cdot Ft}{FH} \quad tu := \frac{Hv}{2}$$

$$Lu := HL - Fu \quad Lt := \sqrt{Lu^2 + tu^2}$$

$$Lg := \frac{Lt \cdot Lf}{Lu}$$



$$MN := \frac{DN}{2} \quad NL := \frac{AN}{2}$$

$$ML := \sqrt{NL^2 - MN^2}$$

$$Mb := ML + Lb$$

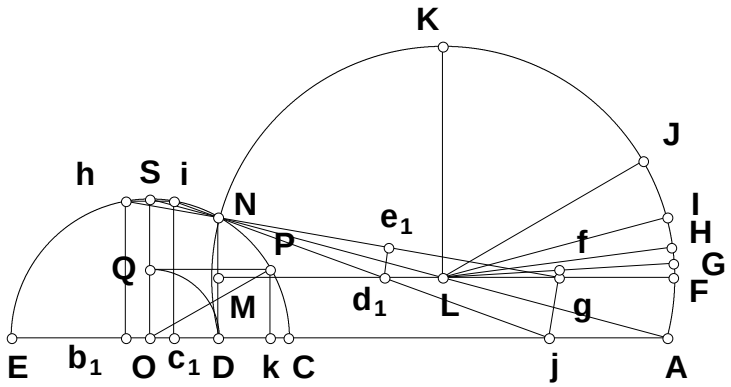
$$Nb := \sqrt{MN^2 + Mb^2}$$

$$Nc := \frac{CO^2 + (2 \cdot Nb)^2 - (2 \cdot Mb + DO)^2}{-2 \cdot Nb} \quad cw := \frac{MN \cdot (Nb + Nc)}{Nb} + MN$$

$$Dw := \frac{2 \cdot Mb \cdot (2 \cdot Nb + Nc)}{2 \cdot Nb} - 2 \cdot Mb \quad Ow := Dw - DO \quad Dx := Dw - 2 \cdot Ow \quad Ma_1 := \frac{Dx \cdot DN}{2 \cdot (cw - DN)}$$

$$Na_1 := \sqrt{MN^2 + (Ma_1)^2} \quad ba_1 := Mb - Ma \quad Nz := \frac{(Na_1)^2 + Nb^2 - (ba_1)^2}{2 \cdot Nb} \quad za_1 := \sqrt{(Na_1)^2 - Nz^2}$$

$$be := \frac{za_1 \cdot Nb}{Nz}$$



$$Mg := ML + Lg \qquad Ng := \sqrt{MN^2 + Mg^2}$$

$$Nh := \frac{CO^2 + (2\cdot Ng)^2 - (2\cdot Mg + DO)^2}{-2\cdot Ng} \qquad hb_1 := \frac{MN\cdot(Ng + Nh)}{Ng} + MN$$

$$Db_1 := \frac{2\cdot Mg\cdot(2\cdot Ng + Nh)}{2\cdot Ng} - 2\cdot Mg \qquad Ob_1 := Db_1 - DO \qquad Dc_1 := Db_1 - 2\cdot Ob_1$$

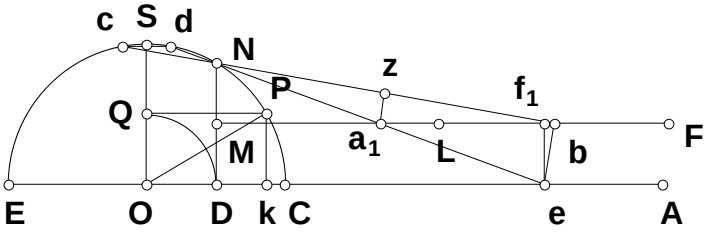
$$Md_1 := \frac{Dc_1\cdot DN}{2\cdot(hb_1 - DN)} \qquad Nd_1 := \sqrt{MN^2 + (Md_1)^2} \qquad gd_1 := Mg - Md_1$$

$$Ne_1 := \frac{(Nd_1)^2 + Ng^2 - (gd_1)^2}{2\cdot Ng} \qquad ed_1 := \sqrt{(Nd_1)^2 - (Ne_1)^2} \qquad gj := \frac{ed_1\cdot Ng}{Ne_1} \qquad Ne := \frac{Na_1\cdot Nb}{Nz}$$

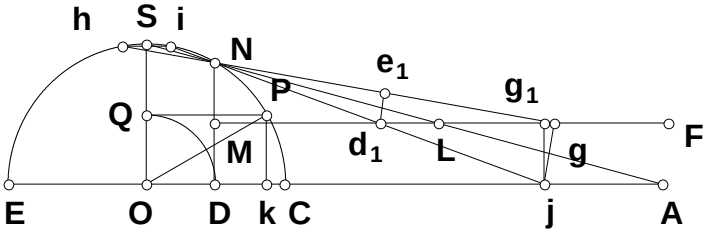
$$ea_1 := Ne - Na_1$$

$$ba_1 := Mb - Ma_1$$

$$bf_1 := \frac{be^2 + ba_1^2 - ea_1^2}{2\cdot ba_1}$$



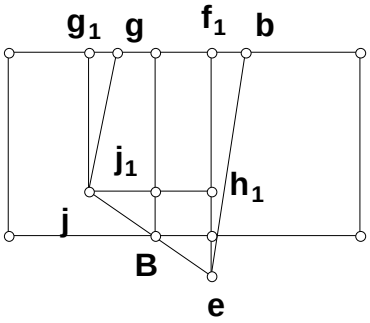
$$ef_1 := \sqrt{be^2 - bf_1^2}$$



$$Nj := \frac{Nd_1\cdot Ng}{Ne_1} \qquad jd_1 := Nj - Nd_1$$

$$gg_1 := \frac{gj^2 + gd_1^2 - jd_1^2}{2\cdot gd_1}$$

$$jg_1 := \sqrt{gj^2 - gg_1^2}$$



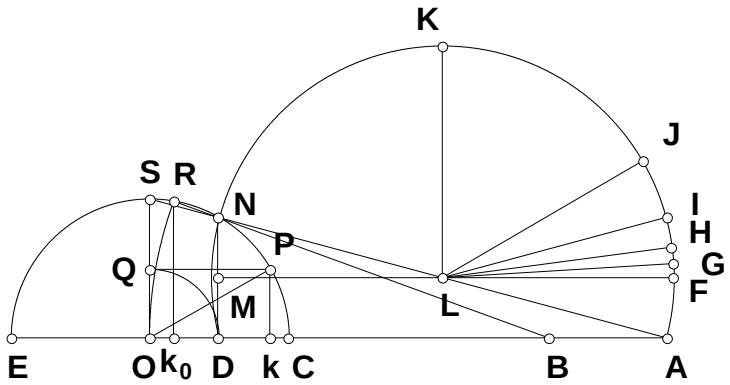
$Mg_1 := Mg - gg_1$ $Mf_1 := Mb - bf_1$

$jh_1 := Mf_1 - Mg_1$ $eh_1 := ef_1 - jg_1$

$ej := \sqrt{jh_1^2 + eh_1^2}$ $Bj_1 := DM - jg_1$

$jj_1 := \frac{jh_1 \cdot Bj_1}{eh_1}$ $BD := Mg_1 + jj_1$

$BO := BD + DO$



$BN := \sqrt{DN^2 + BD^2}$

$NR := \frac{CO^2 + BN^2 - BO^2}{-BN}$

$Bk_0 := \frac{BD \cdot (BN + NR)}{BN}$

$Ok_0 := BO - Bk_0$

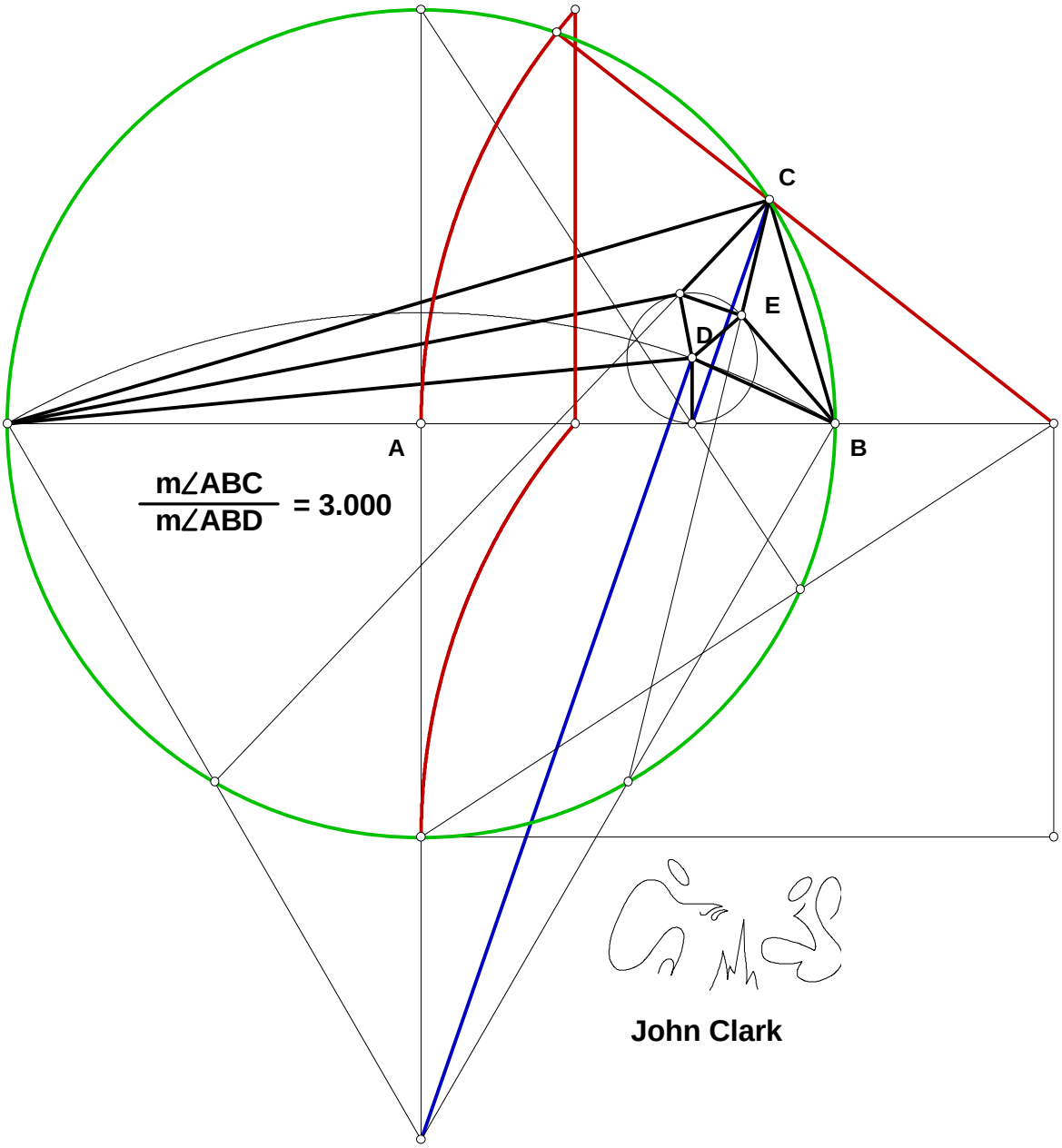
$NR - 2 \cdot Ok_0 = -0.000000000019581$

$Om - \frac{Ek}{2} = -0.000000000002639$ ∎

Compared to 022803

$\frac{19591}{2639} = 7.423645$

Three Pieces Of Paper



John Clark

Three Pieces Of Paper

Introduction

Is a novel about the search for a method to divide angles into at least three equal parts. The novel is written in the relatiologic of Geometry and the tautologic of basic Algebra.

I suspect that someday I might have something of significance to write here. J. C.